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SOURCE RADIATION IN THE PRESENCE OF SMOOTH CONVEX BODIES.(U)

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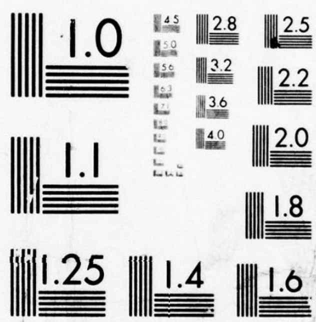

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TECHNICAL REPORT NO. 78-3

June 1978

SOURCE RADIATION IN THE PRESENCE OF SMOOTH CONVEX BODIES

AD No.
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S. Safavi-Naini
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ELECTROMAGNETICS LABORATORY
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The problem of radiation from sources in the presence of smooth, convex, impenetrable objects is considered, and a brief survey of various high frequency techniques is presented. A generalization of the geometrical theory of diffraction and two new techniques, based on the spectral domain approach and an asymptotic evaluation of the radiation integral for the surface current, are also discussed. Some numerical results derived from the spectral domain formulas are presented next page

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SOURCE RADIATION IN THE PRESENCE OF SMOOTH CONVEX BODIES

Technical Report

S. Safavi-Naini and R. Mittra

June 1978

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ABSTRACT

The problem of radiation from sources in the presence of smooth, convex, impenetrable objects is considered, and a brief survey of various high frequency techniqes is presented. A generalization of the geometrical theory of diffraction, and two new techniques based on the spectral domain approach and an asymptotic evaluation of the radiation integral for the surface current, also are discussed. Some numerical results derived from the spectral domain formulas are presented and a comparison with available theoretical and experimental data is included.

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1. Introduction

The problems of radiation from sources in the presence of impenetrable smooth convex objects and the diffraction of a plane wave by such objects are of great practical interest in the design of antennas on structures, e.g., conformal arrays. Unfortunately, the exact analytical solutions to these problems, based on the methods of "separation of variables" or "function-theoretic" procedures (Wiener-Hopf technique, residue calculus, etc.), exist only for a very limited number of scattering geometries. Furthermore, the exact solutions are typically highly complex in nature; hence, the process of extracting numerical results from them can be very time-consuming and is by no means trivial. This situation has motivated many researchers to explore approaches to the problems of radiation and scattering from smooth convex structures.

In the low and resonant frequency ranges, several reliable numerical procedures, e.g., the moment method, are available for solving the radiation and scattering problems. However, in the high frequency domain, numerical techniques based on matrix methods become unwieldly if not impractical, prompting one to employ asymptotic techniques suitable for large $k(=2\pi/\lambda)$, where λ is the wavelength of the illuminating wave.

In this work, we begin by presenting, in Sec. 2 , a survey of various high frequency asymptotic techniques for the problem stated above. The survey will be necessarily brief, and will cover only the highlights of a number of important approaches to the problem at hand, viz., Fock's theory, the geometrical theory of diffraction (GTD), and the direct integral equation approach. The reader interested in further details may choose to consult the works of Bowman, et al. [1], Uslenghi [2], and Kouyoumjian [3].

In Sec. 3, we consider the generalization of GTD and present some new approaches to the curved surface radiation and scattering problems. Some numerical results based on one of these new approaches are presented in Sec. 3 and a comparison with other available methods are included.

2. Survey of Available High-Frequency Asymptotic Techniques

2.1 Watson Transformation

One of the first successful attempts to derive an asymptotic expansion for the far-field generated by a point source located in the proximity of a conducting surface was made by G. N. Watson in 1918 [4]. His method, essentially, consisted of two steps: 1) transforming the original infinite series solution into a contour integral (by Cauchy's residue theorem); 2) deforming the contour of integration so as to capture a set of complex poles of the integrand. The original integral is then expressed in terms of an infinite series which converges very rapidly, provided the observation point is in the shadow region. The first few terms of this series were later interpreted as "creeping waves." The method was first applied to a sphere and circular cylinder, and later to some other geometries as well. The mathematical rigor of the method was the subject of further investigations by other researchers ([5], [6] and [7]). Although Watson transformation can only be applied to a few simple goemetries, e.g., the sphere, cylinder, cone, spheroid, it is still regarded as one of the cornerstones of the more general high frequency techniques because of its mathematical rigor. Watson transformation is especially powerful in the shadow region of the geometric optics field. In the lit region, the above-mentioned contour integral is evaluated using the "stationary phase" method and yields the reflected field from the surface. In this region, the most significant contribution to the total scattered field typically comes from the surface current induced on the smooth convex part of the object; the so-called "Physical Optics" approximation can be applied

([8], [9], and [10]) to derive the reflected field. The Physical Optics method is based upon approximating the induced surface current in the lit region of the object by the current that would be induced on the local tangent plane, and by assuming that the surface current is zero in the shadow region. The far field is constructed by substituting the above estimate for the induced surface current in the integral representation of the scattered field, and evaluating the same in an asymptotic sense. The dominant term of the asymptotic expansion of this integral can be shown to be identical to the first term of the Luneberg-Kline expansion of the geometrical optics far field ([11] and [12]). However, the higher-order terms derived from the physical optics approach do not provide us with the correct result in the shadow or transition regions where the diffracted field contributes the most.

In the next section, we discuss Fock's theory, which can fill the gap between the Physical Optics in the lit region and the "creeping wave" representation in the shadow region.

2.2 Fock's Theory

The region between the lit and the shadow part on a surface is called "penumbra region." The angular width of this region is approximately given by $(\lambda r_o^2/\pi)^{1/3}$ where λ is the wavelength of the illumination and r_o is the radius of curvature of the surface of the object in this region in the incident plane (Fig. 1). Fock's theory invokes the principle of *local* character of the field in the penumbra region [13] and is based on the conjecture that all bodies with a smoothly varying curvature have the same current distribution in the penumbra region, provided that the curvature and the incident wave are the same near the point under consideration. This principle allows one to locally replace the surface of the object by a portion of a paraboloid of

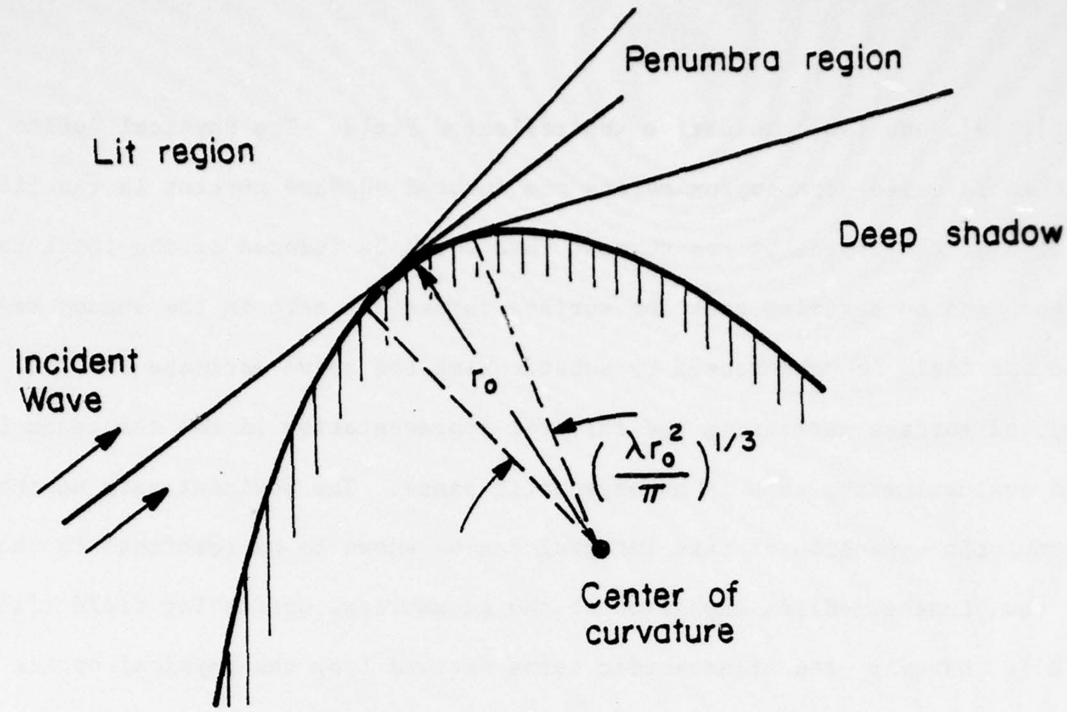


Figure 1: Section of the body in the plane of incidence.

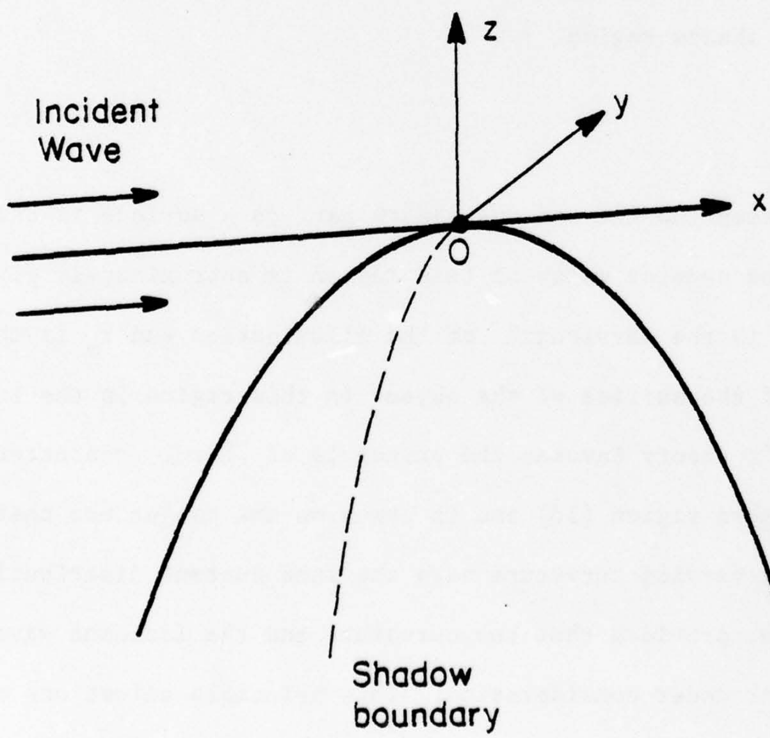


Figure 2: Plane wave incident upon a smooth convex body.

revolution. A unique feature of the expressions for Fock currents is that they provide a convenient transformation of the geometric optics currents in the lit region into the creeping wave currents in the shadow region. Fock himself deduced the pertinent formulas for the surface currents by treating a convex body problem [14] described below.

Consider a convex body and a plane wave incident in the direction of the x-axis. If the equation of the surface is

$$f(x,y,z) = 0 \tag{1}$$

then the curve representing the boundary of geometrical shadow is given by

$$f(\vec{r}) = 0 \ , \quad \frac{\partial f}{\partial x} = 0 \tag{2}$$

Consider a point 0 on the boundary of a shadow region where we set up a rectangular coordinate system as shown in Fig. 2 ($\hat{z}$: normal to the surface, $\hat{x}$: in the direction of propagation, and $\hat{y}$ is the tangent to the boundary of shadow). In the vicinity of this point, the surface of the body could be locally replaced by a paraboloid of revolution which is expressed by the equation.

$$z + 1/2\ (ax^2 + 2bxy + cy^2) = 0 \tag{3}$$

Each of the field components satisfies the Helmholtz equation

$$(\nabla^2 + k^2)\Psi = 0 \tag{4}$$

The fact that the incident wave travels along the x-axis, suggests that Ψ be written in the form

$$\Psi = \tilde{\Psi} e^{-jkx} \tag{5}$$

where an $\exp(j\omega t)$ time dependence has been assumed. Substituting (5) in (4) gives

$$\nabla^2 \tilde{\Psi} - 2jk \frac{\partial}{\partial x} \tilde{\Psi} = 0 \tag{6}$$

At this point, two basic assumptions are introduced in Fock's theory, viz.

i) $\tilde{\Psi}$'s are relatively slowly varying function of coordinates

ii) $\tilde{\Psi}$ varies more rapidly in the z-direction than in x and y, i.e.,

$$\frac{\partial \tilde{\Psi}}{\partial z} = 0(\frac{k}{m} \tilde{\Psi}), \; \frac{\partial \tilde{\Psi}}{\partial x} = 0(\frac{k}{m'}, \tilde{\Psi}), \; \frac{\partial \tilde{\Psi}}{\partial y} = 0 \; (\frac{k}{m} \tilde{\Psi}) \tag{7}$$

Based upon (7), we can write (6) as

$$\frac{\partial^2 \tilde{\Psi}}{\partial z^2} - 2jk \frac{\partial \tilde{\Psi}}{\partial x} = 0 \tag{8}$$

and consequently $m' = m^2$ (m is very large), where the terms of order $1/m^2$ have been omitted.

Inserting these estimates and assumptions into the Maxwell's equation, we can find some simple expressions for all the field components in terms of H_y and H_z. If we write H_y as

$$H_y = H_y^0 \, e^{-jkx} \, \tilde{\Psi} \tag{9}$$

where H_y^0 is the magnitude of the incident wave at infinity, then Ψ must satisfy

$$\frac{\partial^2 \tilde{\Psi}}{\partial z^2} - 2jk \frac{\partial \tilde{\Psi}}{\partial x} = 0 \tag{10}$$

with boundary condition

$$\frac{\partial \tilde{\Psi}}{\partial z} - jk \left(ax + by + \frac{1}{\sqrt{\eta}} \right) \tilde{\Psi} = 0 \tag{11}$$

on the surface of the body. Eqn. (11) is the simplified version of the Leontovich boundary condition where

$$\eta = \varepsilon - j \frac{4\pi\sigma}{ek}$$

The final solution for H_y on the surface of the body which satisfies the boundary condition and the condition at infinity, may be written in the form

$$H_y = H_y^{ex} \, G(\xi, q) \tag{12}$$

where H_y^{ex} = external field

$$G(\xi, q) = e^{-(j/3)\,\xi^3} \, V_1(\xi, q)$$

$V_1(\xi, q)$ = Fock function defined in the Appendix A.

$\xi = m(ax + by)$ = reduced distance from the shadow boundary = ℓ/d.

d = the width of penumbra region = $(2r_o{}^2/k)^{1/3}$

ℓ = distance between the observation point and the shadow boundary along the incident ray (Fig. 3).

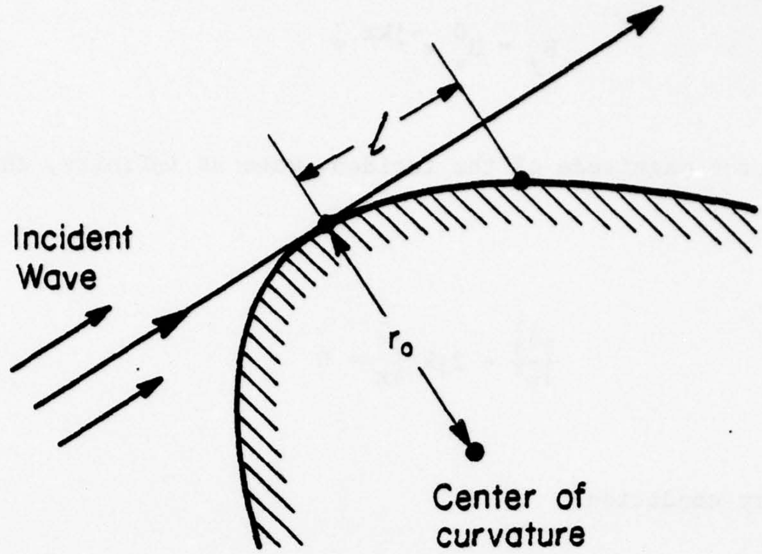


Figure 3: Geometric meaning of the quantity ℓ in (12).

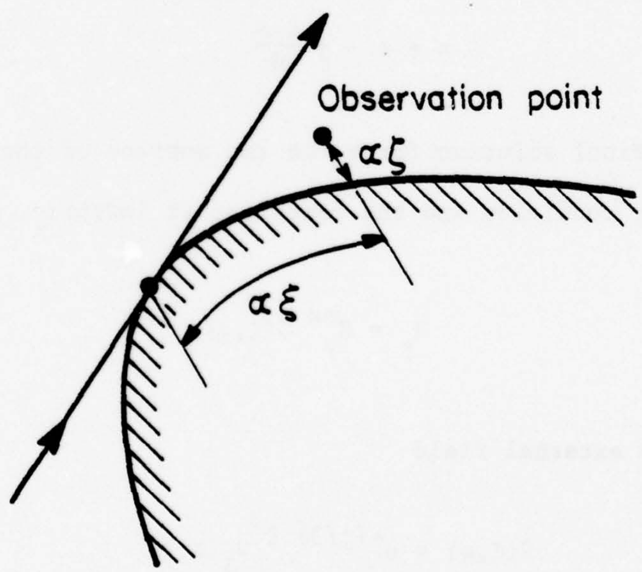


Figure 4: Coordinates of observation point in terms of ξ and ζ.

$q = -jm/\sqrt{\eta} = -(j/\eta)\sqrt[3]{\frac{k}{2a}}$ (= 0 for conducting body)

The other tangential component of the magnetic field H_x on the surface of the body can be obtained in a similar manner

$$H_x = H_z^{ex}\,[-\frac{j}{m}\; e^{-(j/3)\xi^3/3}\; f(\xi)] \tag{13}$$

where $f(\xi)$ is another Fock function defined in Appendix A. Fock's formulas not only give the surface value of the field, but also can be utilized to find the field in the proximity of the object. For a plane wave incidence, the first order, i.e., $0(1/m)$ terms for the scattered field within a certain layer around the object, can be written as

$$\begin{aligned} &H_x = 0,\quad H_y = H_y^0\; e^{-jkx}\,\tilde{\Psi}\,(\xi,\zeta),\quad H_z = H_z^0\; e^{-jkx}\,\tilde{\Phi}(\xi,\eta) \\ &E_x = (j/m) H_y^o\; e^{-jkx}\; \partial\tilde{\Psi}/\partial\zeta,\quad E_y = H_z,\quad E_z = -H_y \end{aligned} \tag{14}$$

where

$\zeta = 2am^2[z + (1/2)(ax^2 + 2bxy + cy^2)]$ = reduced height from the surface of the body (Fig. 4).

$$\begin{aligned} &\tilde{\Psi} = -je^{(j\xi\zeta - j/3\;\xi^3)} \int_c e^{-j\xi t}\; [w_1(t-\zeta) - \frac{w_1'(t)-qw_1(t)}{w_2'(t)-qw_2(t)}\, w_2(t-\zeta)]\; dt \\ &\tilde{\Phi} = \frac{-je^{j\xi\zeta-(j/3)\xi^3}}{2\sqrt{\pi}} \int_c e^{-j\xi t}\; [w_1(t-\zeta) - \frac{w_1(t)}{w_2(t)}\, w_2\;(t-\zeta)]\; dt \end{aligned} \tag{15}$$

The path of integration for $\tilde{\Phi}$ and $\tilde{\Psi}$ is shown in Fig. 5 .

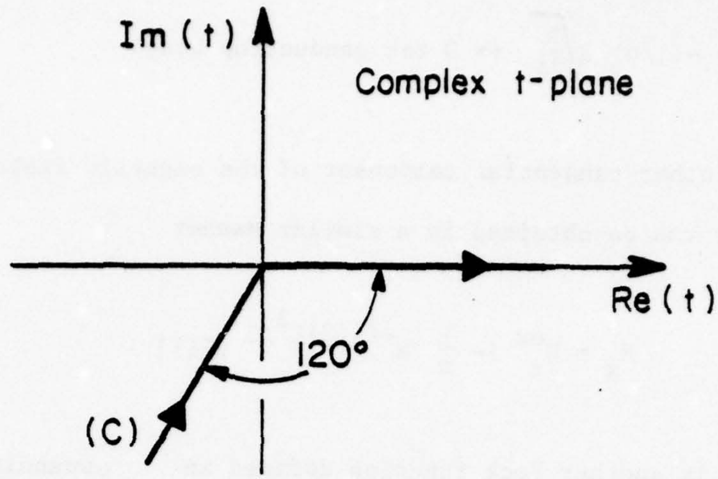


Figure 5: Path of integration for (15) in the complex t-plane.

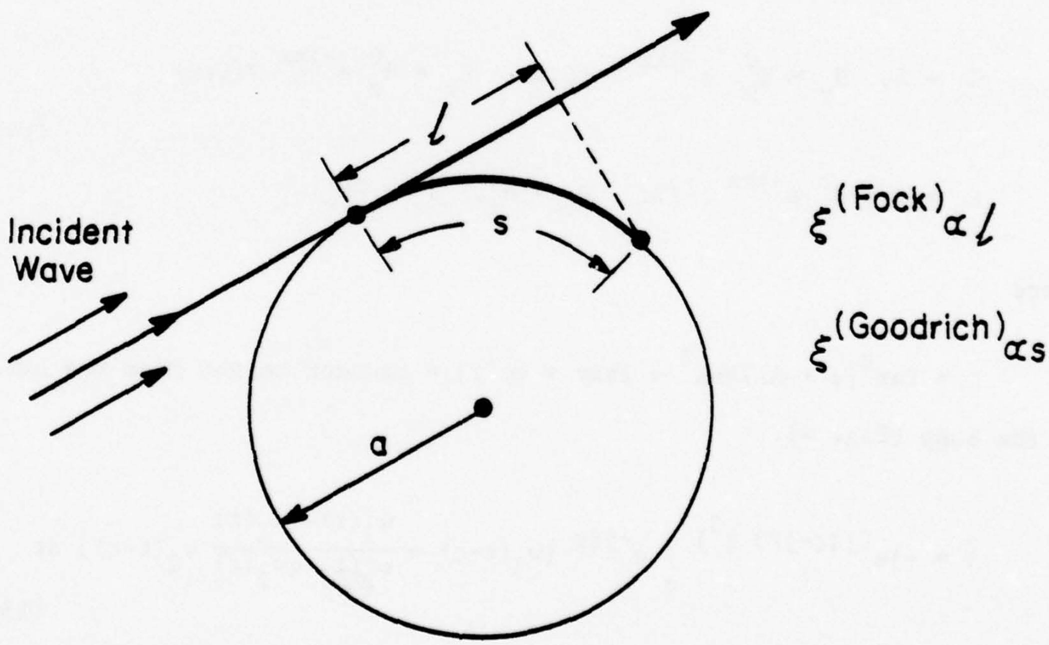


Figure 6: Comparison between the various definitions of parameter ξ for the case of a circular cylinder.

Fock's expressions for the field components in the penumbra region ($\xi \simeq 0$) can be extended to the shadow region, by introducing some modifications in the definition of parameter ξ. Goodrich [15] has generalized the argument used by Fock in the penumbra region to anywhere in the shadow region by introducing a new set of variables, ξ and ζ, for the incremental distances along the path leading into the shadow region. In this generalization, the parameter ξ as defined in (12) is replaced by

$$\xi = \int_0^s \left(\frac{kR(s)}{2}\right)^{1/3} \frac{ds}{R(s)} \tag{16}$$

where s is the arc length along the geodesics which originate from the shadow boundary and go into the shadow region along the surface, and R(s) is the radius of curvature of the surface along the geodesics. For the case of a circular cylinder of radius a (Fig. 6), the expression ξ simplifies to

$$\xi = (ka/2)^{1/3} \theta = s/d \tag{17}$$

Fock also treated the case where the point source was very close to the surface of the body. He analyzed the radiation of electric dipoles near a spherical model of the earth [16] and derived the formulas for the scattered fields in terms of functions (attenuation functions) similar to $\tilde{\phi}$ and $\tilde{\Psi}$, which are valid both in the shadow and transition regions [17]. Fock's assumptions were later proven in a more systematic and mathematically rigorous manner by Cullen [18] and Hong [19] by using a direct integral equation approach. This method is described in the next section.

2.3 Direct Integral Equation Approaches

This method, which is closely related to Fock's theory, can be illustrated by analyzing the diffraction of a plane electromagnetic wave by an arbitrary conducting body (large compared with λ). Cullen [18] obtained a first-order asymptotic solution to the integral equation for the induced surface current

$$\vec{J}(\vec{r}) = 2\vec{n}(\vec{r}) \times \vec{H}^{inc}(\vec{r}) - (1/2\pi)\vec{n}(\vec{r}) \times \int_S \int ds' \frac{1+jkR}{R^3} \{\vec{J}(\vec{r}') \times \vec{R}r^{-jkR}\} \tag{18}$$

where $\vec{n}(\vec{r})$ is the outward unit normal to the surface at $\vec{r}$, $\vec{H}^{inc}(\vec{r})$ is the incident magnetic field on the surface (S) of the body, and $R = \vec{r} - \vec{r}'$ ($\vec{r}'$ is a variable point on the surface).

Fock used this integral equation to deduce the important principle of local character of the field in the penumbra region. Cullen derived a first-order asymptotic solution to (18) which agreed with Fock's results given in (12) and (13). Cullen's method consists of transferring the two-dimensional integral equation (18), in the penumbra region, to a one-dimensional, Volterra-type equation. This is accomplished by applying the stationary phase technique to the original integral while integrating with respect to one of the variables. The resulting one-dimensional Volterra equation is then solved in Cullen's method by the Fourier transform technique. A similar procedure was used by Hong [19] to analyze, asymptotically, the diffraction of electromagnetic and acoustic plane waves by smooth convex bodies. We will now proceed to explain Hong's method in a little more detail by referring back, once again, to the integral equation (18). The surface is parametrized by the geodesic coordinate system (σ,v) such that the shadow boundary for the incident plane wave traveling along the tangent $\hat{\sigma}(0,v)$ to the $v = 0$ curve is the $\sigma = 0$ curve. The quantities $\hat{\sigma}(\sigma,v)$, $\hat{b}(\sigma,v)$ and $\hat{n}(\sigma,v)$ form a right-hand local orthonormal basis $(\hat{n} = \hat{\sigma} \times \hat{b})$ (Fig. 7).

Since the incident field has a phase factor $e^{-jk\hat{\sigma}(0,0)\cdot\hat{r}(\sigma,0)}$, we write the surface current in the form

$$\vec{J}(\vec{r}) = [I_\sigma(\vec{r})\ \hat{\sigma}(\vec{r}) + I_b(\vec{r})\ \hat{b}(\vec{r})]\ e^{-jk\sigma} \tag{19}$$

where σ is the arc length along the geodesic. Substituting (19) back into (18) and restricting the resulting equation to the points on the geodesic $v=0$, we obtain two coupled, two-dimensional integral equations for $I_\sigma(\sigma,0)$ and $I_b(\sigma,0)$. It can be shown that these integrals have saddle points at $v=0$ (for the v-integration). Applying the "steepest descent path" method to v-integration, and keeping the terms up to the order $1/M_0^2$, where $M_0 = (k\rho_\sigma\ (\sigma,0))^{1/3}$, we obtain the following decoupled one-dimensional, Volterra-type integral equations for $I_b(\xi,0)$ and $I_\sigma(\xi,0)$

$$\begin{aligned} I_\sigma(\xi,0) &= 2\ I_\sigma^{inc}(\xi,0) - \int_{-\infty}^{\xi} d\tau\ I_\sigma(\xi,0)\ K_\sigma(\xi-\tau) + 0\,(M_0^{-3}) \\ I_b(\xi,0) &= 2\ I_b^{inc}\ (\xi,0) - \int_{-\infty}^{\xi} d\tau\ I_b(\xi,0)\ K_b(\xi-\tau)\ + 0(M_0^{-3}) \end{aligned} \tag{20}$$

$\rho_\sigma(\sigma,v)$ is the radius of curvature of the surface along geodesics (v = constant curves) at point (σ,v).

Solving (20) by Fourier transforms, we obtain the expression for the induced currents in the penumbra and shadow regions, and the first-order solutions

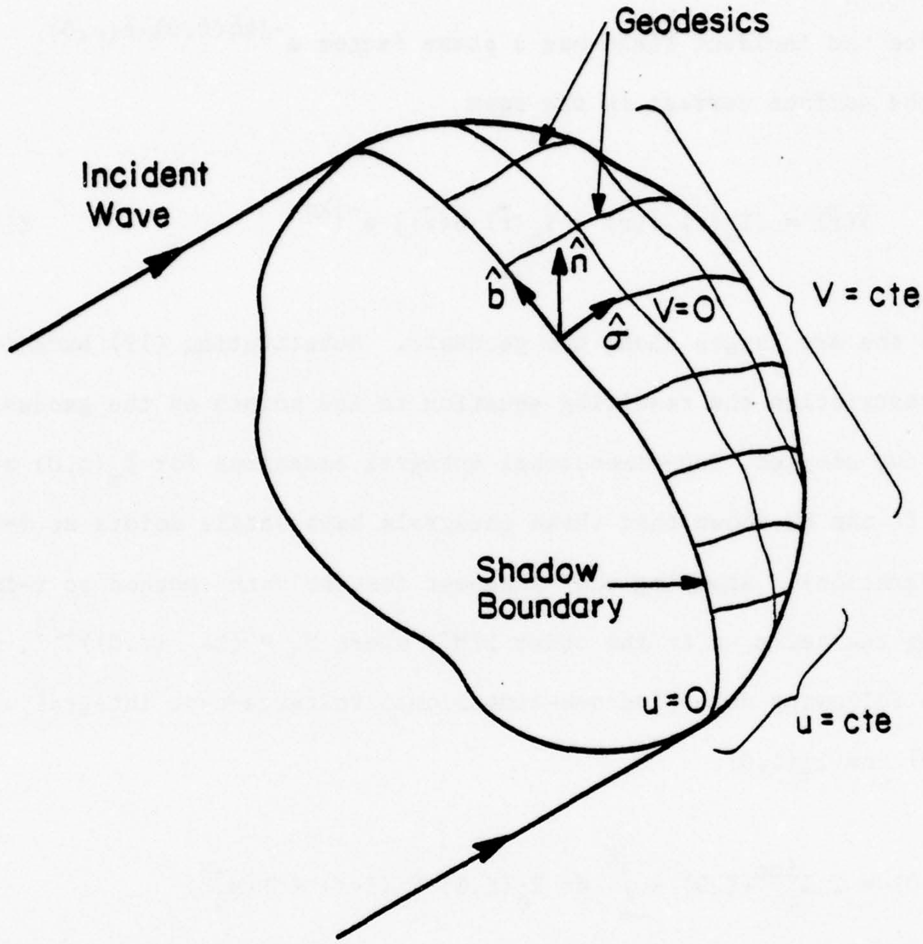


Figure 7: Geodetic coordinate system on a smooth convex body.

are found to be the same as those of Fock and GTD [20]. One of the important conclusions drawn from Hong's solution is that the leading term in the asymptotic expansion, which is the same for the acoustic and electromagnetic problems, is independent of curvature in the direction transverse to the geodesic, provided the divergence factor is suppressed. However, we should bear in mind that Hong's method was designed for the case of axial incidence on symmetric objects, and in this case, the geodesics are torsionless. The above conclusion does not seem to be valid in the cases where the rays have nonzero torsion ([21], [22]). In Hong's expressions for the surface current, the transverse curvature has only a second-order effect. It was also shown that up to the terms of order $(k\rho_\sigma)^{-2/3}$ in the asymptotic expansion, the tangential and binormal components of the creeping waves are not coupled.

Both Fock's theory and the "direct integral equation approach" give the induced surface current, or the scattered field in the neighborhood of the surface of the scatterer, due to an incident plane wave. These expressions can also be used to derive the radiated field via the use of the reciprocity theorem.

The methods which have been discussed thus far are mathematically rigorous. However, they are limited in the scope of their application to geometries satisfying some special smoothness and symmetry criteria. "Geometrical theory of diffraction" (GTD), which we discuss in the next section, has a broader scope, although it does lack the mathematical rigor of approaches described until now.

2.4 Geometrical Theory of Diffraction (GTD)

Geometrical theory of diffraction (GTD), developed by J. B. Keller ([20], [23], [24], [25], and [26]), is a generalization of geometrical optics.

It is based upon the assumption that fields propagate along rays. Keller's major contribution was to introduce the new kinds of rays called the "diffracted rays," which together with the geometrical optics rays, constitute the total field. In our problem, viz., source radiating in the proximity of the smooth object, the diffracted rays travel along the curves on the surface of the scatterer. By applying Fermat's principle to these surface rays, we conclude that the above-mentioned curves should be geodesics on the surface of the body. In the GTD procedure, one assigns a value to the field along each ray of these surfaces. The total field at any point in the space is the sum of the fields due to various rays (incident, reflected and diffracted) passing through that point. An important advantage of the GTD approach is that it can be applied to both scalar (acoustic) and vector (electromagnetic) problems and to smooth convex objects of an arbitrary shape.

Consider the problem of determining the radiated field of a scalar point source located on the surface of a smooth convex opaque body. If the observation point is in the shadow region, the ray paths originating at Q and reaching P (observation point) are comprised of two sections. One of these sections follows the straight line path P_1P, while the other travels along a geodesic on the surface (Fig. 8). Let us consider the propagation of the field along each section separately.

a) Rays in free space: Behavior of the fields along these rays can be determined by obtaining a high-frequency asymptotic solution to Maxwell's equation in a source-free homogeneous isotropic medium. We begin with the Luneberg-Kline asymptotic expansion of the electric field ([11] and [12]):

$$\vec{E}(\vec{r}) \sim k^{\tau} e^{-jk_o S(\vec{r})} \sum_{m=0}^{\infty} (jk)^{-m} \vec{e}_m(\vec{r}) \tag{21}$$

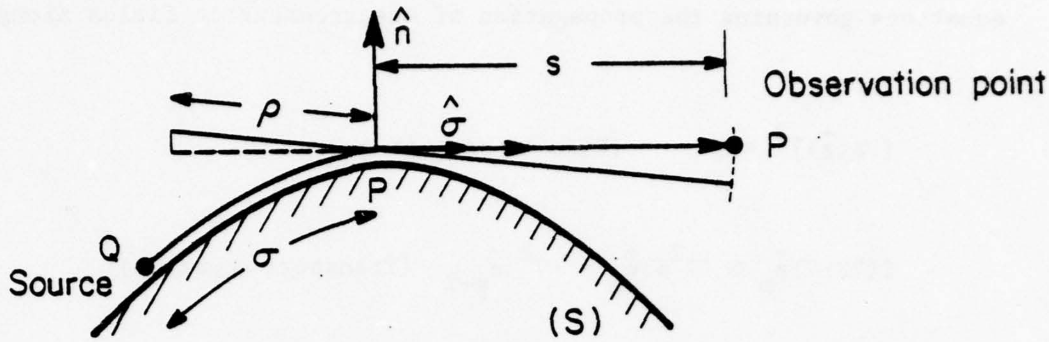


Figure 8: Diffraction by a smooth convex body when the observation point is in the shadow region of the source Q.

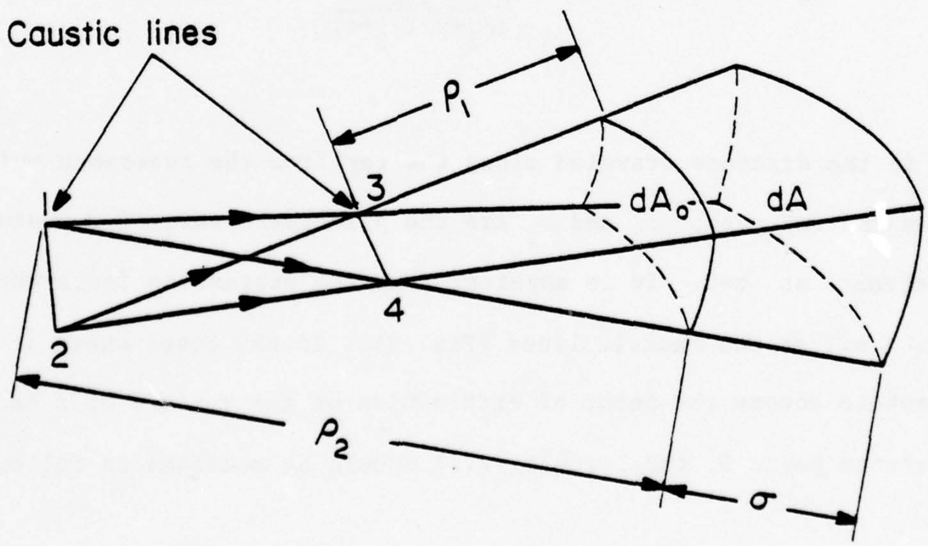


Figure 9: Diverging pencil of rays in free space.

and insert it into the Maxwell's equations. This results in the following equations governing the propagation of electromagnetic fields along the rays.

$$[\nabla S(\vec{r})]^2 = 1 \qquad \text{(Eikonal equation)} \tag{22}$$

$$2(\nabla S\cdot\nabla)\vec{e}_m + (\nabla^2 S)\vec{e}_m = -\nabla^2\,\vec{e}_{m-1} \qquad \text{(Transport equation)} \tag{23}$$

$$\nabla S\cdot\vec{e}_m = -\nabla\cdot\vec{e}_{m-1} \qquad \text{(Gauss's Law)} \tag{24}$$

$$e_{-1} = 0, \qquad m = 0, 1, 2, \ldots$$

The zeroth-order solution to the above system of equations,which turns out to be in agreement with what one would obtain by geometric optics, may be written as

$$E(\sigma) \sim E_0(0)e^{-jk_0 S(0)} \cdot \sqrt{\frac{\rho_1\rho_2}{(\rho_1+\sigma)(\rho_2+\sigma)}}\; e^{-jk_0\sigma} \tag{25}$$

where σ is the distance traveled along the ray from the reference point $0(\sigma=0)$ on the ray path. ρ_1 and ρ_2 are the principal radii of curvature of the wavefront at $\sigma=0$. It is apparent that the expression fails when $\sigma=-\sigma_1$ or $\sigma = -\rho_2$, i.e., at the caustic lines (Fig. 9). In the cases where it is convenient to choose the point of diffraction on the surface of a body as the reference point 0, the formula (2.5) should be modified as follows

$$E(\sigma) \sim \vec{\delta}_0 \cdot \sqrt{\frac{\rho}{\sigma(\rho+\sigma)}}\; e^{-jk\sigma} \tag{26}$$

In these cases, the point of diffraction itself is a caustic, and ρ is the distance between this point and the second caustic.

b) Surface Rays: These rays follow the surface S along the geodesics into the shadow region, and shed off energy tangentially as they propagate. In order to study the behavior of the field along these rays, we introduce a special ray-fixed coordinate system, $\hat{\sigma}, \hat{n}, \hat{b}$.

$\hat{\sigma}$: Unit vector tangent to the ray; $\hat{n}$: outward unit normal to the surface; and $\hat{b} = \hat{t} \times \hat{n}$ or binormal direction; a vector field can be decomposed into its components along these unit vectors as

$$\vec{E} = E_\sigma \hat{\sigma} + E_n \hat{n} + E_b \hat{b} \tag{27}$$

At this point, several important assumptions are introduced in the GTD approach [20]:

i) $\vec{E}$ and $\vec{H}$ are orthogonal to each other and to the ray.

ii) Variation of the phase of the field along the ray is the same for both fields.

iii) E_n and E_b propagate independently, and $E_\sigma = 0$.

iv) E_b satisfies the scalar wave equation $(\nabla^2+k^2)\, u=0$ with the boundary condition $u=0$ on the surface S, while E_n satisfies the same equation with the boundary condition $\frac{\partial u}{\partial n} = 0$.

The next step in the GTD approach is to conjecture, on the basis of the solution to some canonical problems, that the surface field propagating along each ray is comprised of an infinite set of "modes." Along a ray-fixed path GTD assigns a complex value to each component of the field associated with the individual modes. The propagation of these modal field is described by the equation

$$a(\sigma) = A(\sigma)\, e^{j(\phi_0 - k\sigma)} \qquad (28)$$

when σ is the distance between an arbitrary point along the ray and the source Q and ϕ_0 is the phase of the field at the source point. Next, invoking the principle of conservation of energy between two adjacent rays, and using the fact that the surface rays shed energy off tangentially, we can arrive at the following expression for $a(\sigma)$

$$a(\sigma) = K\sqrt{\frac{d\Psi_1}{\rho d\Phi_2}}\ \exp[-jk\sigma - \int_0^{\sigma} \alpha\ (\sigma')d\sigma'] \qquad (29)$$

where $\alpha(\sigma)$ is the "attenuation constant," K is proportional to the strength of the source, and $d\Psi_1$, $d\Psi_2$ and ρ are shown in Figure 10. The quantity $[d\Psi_1/(\rho d\Psi_2)]^{1/2}$ indicates the spreading of the surface ray tube" as it travels along the surface. Equations (26) and (29) describe the laws of propagation for the rays which originate from the source point Q, are diffracted at P_1, and reach the observation point P. To complete the solution, we need to determine the actual values of the fields from these equations. These require the knowledge of δ_0 and K, which, in turn, are related to the initial values of the rays QP_1 and P_1P as well as the attenuation constant $\alpha(\sigma)$. The initial value of the field at Q is related to the strength of the source by L(Q), the so-called "launching coefficient," while the initial value of the field at P_1 is related to the actual field on the surface at P_1 through the "diffraction coefficient" $D(P_1)$. If we now sum up the contributions of all the modes, we obtain the final solution [25] for the field radiated in the shadow region by an infinitesimal magnetic dipole of strength $\vec{M}$ located on a smooth convex conducting body

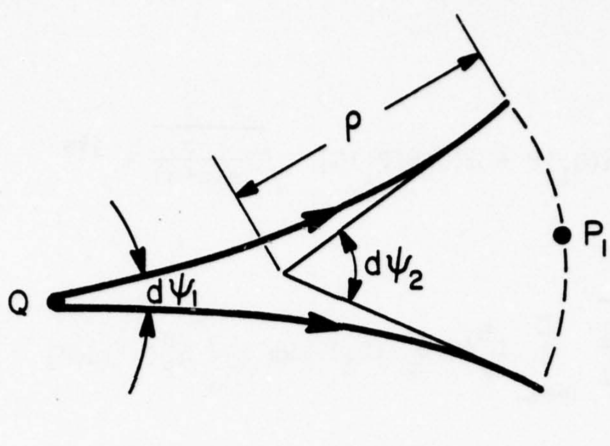


Figure 10: Divergence of surface rays.

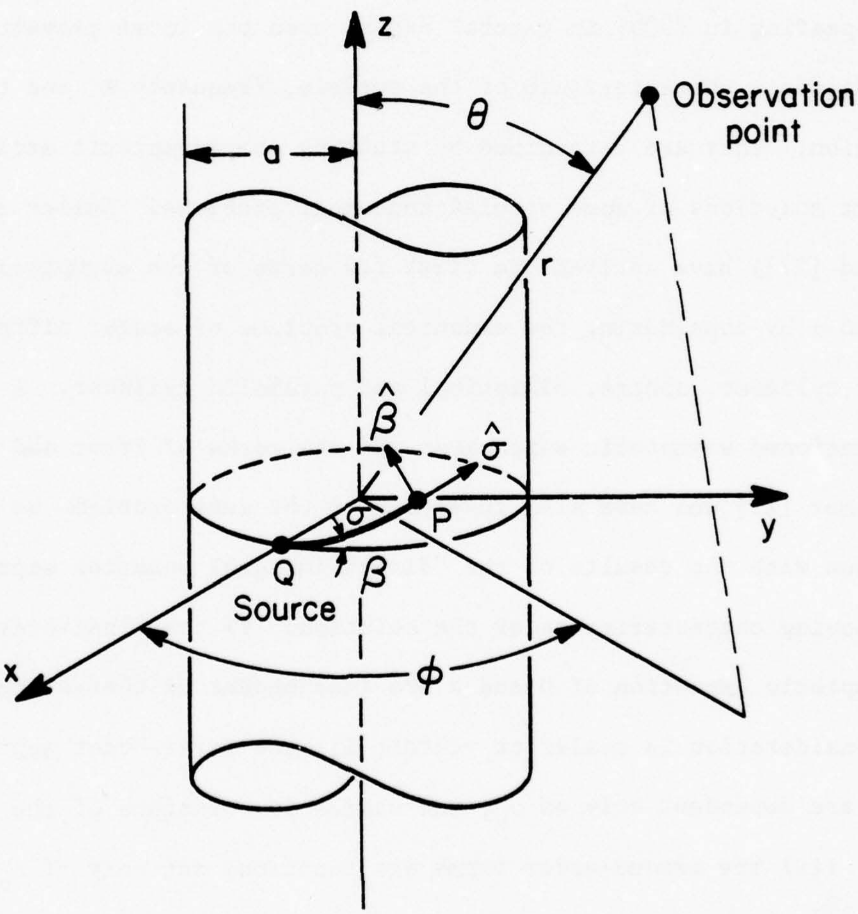


Figure 11: Geometry of the cylinder problem.

$$\vec{E}^{d}(P) = \vec{M} \cdot [\hat{b}(Q)\hat{n}(Q_1)F + \hat{\sigma}(Q)\hat{b}(P_1)G] \sqrt{\frac{\rho}{s(\rho+s)}}\, e^{-jks} \tag{30a}$$

where

$$F = \frac{-jke^{-jk\sigma}}{4\pi}\sqrt{\frac{d\Psi_1}{\rho d\Psi_2}} \sum_{p=1}^{\infty} L_p^h(Q) D_p^h (P_1) \exp\left[-\int_0^{\sigma} \alpha_p^h(\sigma')d\sigma'\right] \tag{30b}$$

and the expression for G is obtained by replacing the superscript "h" by "s" in (30b), where h and s stand for hard and soft boundary conditions, viz., u=0 and $\partial u/\partial n = 0$, respectively. The quantities $L_p^{h,s}$, $D_p^{h,s}$ and $\alpha_p^{h,s}$, appearing in (30b), in general depend upon the local geometry and the electromagnetic characteristic of the surface, frequency k, and the mode of propagation. They are determined by studying the asymptotic expansions of the exact solutions of some special canonical problems. Keller and Levy ([20] and [27]) have derived the first few terms of the asymptotic expansions for D and α by considering the canonical problems of scalar diffraction by a circular cylinder, sphere, elliptical and parabolic cylinder. A study of the above-mentioned asymptotic expansions and the works of Franz and Klante [28] and Voltmer [29], who have also investigated the same problem, as well as a comparison with the results of the "direct integral equation approach," reveals the following characteristics of the solution: i) the first-order terms in the asymptotic expansion of D and α are independent of whether the problem under consideration is scalar or vector; ii) the first-order approximation of D and α are dependent only on ρ_σ, the radius of curvature of the surface along the ray; iii) the second-order terms are functions not only of ρ_σ, but also of $\frac{d^2\rho_\sigma}{d\sigma}$, $\frac{d^2\rho_\sigma}{d\sigma^2}$, and $\rho_{\sigma n}$ (the radius of the curvature of the surface transverse to the ray). Finally, the higher-order terms are different for scalar and vector problems.

The leading terms in the asymptotic expansion of "diffraction coefficient" D, "attentuation constant" α and "launching coefficient" L are presented below:

"Soft" polarization:

$$[D_p^s]^2 = \frac{\pi^{1/2} \cdot 2^{-5/6} \cdot \rho_\sigma^{1/3} \cdot e^{-j\pi/12}}{k^{1/6} \cdot [Ai'(-r_p)]^2} \tag{31}$$

$$\alpha_p^s = \frac{r_p \cdot e^{j\pi/6}}{\rho_\sigma} \left(\frac{k\rho_\sigma}{2}\right)^{1/3} \tag{32}$$

$$L_p^s = e^{-j\pi/12} (2\pi k)^{1/2} \left(\frac{2}{k\rho_\sigma}\right)^{2/3} \cdot Ai'(-r_p) \cdot D_p^s \tag{33}$$

"Hard" polarization:

$$[D_p^h]^2 = \frac{\pi^{1/2} \cdot 2^{-5/6} \cdot \rho_\sigma^{1/3} \cdot e^{-j\pi/12}}{k^{1/6} \cdot r_p' \cdot [Ai(-r'_p)]^2} \tag{34}$$

$$\alpha_p^h = \frac{r_p' \cdot e^{j\pi/6}}{\rho_\sigma} \left(\frac{k\rho_\sigma}{2}\right)^{1/3} \tag{35}$$

$$L_p^h = e^{j\pi/12} \cdot (2\pi k)^{1/2} \left(\frac{2}{k\rho_\sigma}\right)^{1/3} \cdot Ai(-r_p') \cdot D_p^h \tag{36}$$

where Ai(x) is the Airy function:

$$Ai(x) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt \quad (x \text{ real}) \tag{37}$$

and $Ai(-r_p) = 0$, $Ai'(-r_p') = 0$, (Ai' is the derivative of Ai with respect to its argument). Higher-order terms in the expansion of D, α and L have been given in [25] and [30] and in some of the other works on GTD mentioned earlier.

The expression (30) is convenient to use in the lit region. In the shadow part of the transition region, since the exponential decay of the terms in (30) is weak. The convergence of the series representation is very slow. Furthermore, the series diverges in the lit part of the transition region. Consequently, in these regions, it is more reasonable to use an integral representation for the surface ray field, which, in our case, can be expressed in terms of Fock functions [25].

Attempts have been made to establish the mathematical validity of GTD and to minimize its "nondeductive parts" (parts which are based upon physical intuition or the study of the asymptotic solution of some simple problem geometrical concepts of different kinds of rays, diffraction coefficients, attenuation constants, etc.). Kravtsov [31] and Ludwig [32] have analyzed the field near the caustic surface (smooth envelope of a family of ray), and have developed a "uniform asymptotic solution" in the sense that it is finite at the caustic and reduces to geometrical optics away from the caustic.

3. Generalization of GTD and Investigation of Alternate Methods

3.1 Generalization of GTD to Arbitrary Surfaces

Keller's generalization of GTD for the analysis of the field diffracted from a smooth convex object is closely related to what is known as the "boundary layer technique" in the theory of differential equations [43]. On the other hand, the "uniform asymptotic theory" is analogous to the method used by R. E. Langer and F. J. Oliver to find the asymptotic solutions of the second-order differential equations near their "turning points," which are counterparts of the transition regions in our case [33], [34], and [35].

The procedure is based upon the generalization of the geometric optical interpretation of the circular cylinder problem. The solution obtained by this method involves some functions with unknown phase and amplitude, similar to Bessel and Hankel functions. Since the surface of a smooth object is actually the caustic surface of diffracted rays, the above-mentioned formulation is applicable in this case, too. Lewis, et al. [36] have modified this solution to make it satisfy the boundary condition on a convex body. Using ray formalism, they have obtained an asymptotic solution in a complicated form, which they call "creeping wave" and satisfies the boundary condition on and is uniformly valid near and away from the surface. It should be mentioned that the method has been developed primarily for scalar diffraction problems.

Creeping waves that are traveling on the surface of the body generate other kinds of diffracted rays in the presence of the irregularities in the geometric or electromagnetic characteristics of the surface. The effects of discontinuity in the surface curvature, its higher-order derivatives, or the surface impedance have been studied by many authors [37], [38], [39] or [40]. An exhaustive study of various diffraction mechanisms and corresponding diffraction coefficients, and constants associated with the propagation of creeping waves, has been carried out by Albertsen [41].

At this point, let us examine one of the most important features of GTD and its various modifications. GTD formulation is essentially scalar in nature and is heuristic in some parts. Thus, when GTD is applied to a vector problem, it is not surprising that the coupling between various components of the fields are neglected, and each one of them is treated as an uncoupled scalar wave. The other assumptions in GTD are concerned with the directions of these field components and the kind of boundary conditions they satisfy (see Sec. 2.4). As mentioned earlier, non-deductive parts of GTD are based on asymptotic expansions of known solutions to some selected "canonical" problems. Quite often these canonical problems are not general enough to fully and accurately describe the local behavior of the field for an arbitrary structure. Finally, most of the canonical problems investigated are two-dimensional in nature. The only exception to this is the sphere. However, in so far as the geometric properties of the surface are concerned, the sphere is a very special case since its radius of curvature is the same in all directions and, consequently, the surface rays are torsionless. Finally, GTD fails when the observation point is located in the transition regions, shadow boundaries or in the neighborhood of a caustic. In each of these regions, one needs to carefully modify the GTD formulas and often such a modification is not too simple. Nevertheless, in spite of these difficulties, GTD is recognized to be a powerful high-frequency technique for computing the leading terms of the asymptotic solution. Two of the principal attributes of GTD are its simplicity and wide scope of application.

3.2 Spectral Domain Approach

We now examine an approach different from GTD which uses the spectrum of the induced current, or the expression for the radiated field, as a starting point. In order to gain a better insight into the curved-surface radiation and scattering problem and to verify the basic assumption of GTD, it is worthwhile to consider such alternative approaches, particularly if they apply to canonical problems which are more general in nature than those employed to derive the GTD results. An example of such a study would be to consider the case of surface ray propagation with non-zero torsion, a situation that occurs when a magnetic dipole source radiates from a location on the surface of a circular cylinder.

The geometry of the problem is shown in Fig. 11 (p. 21). The radius of the cylinder is a and the source, which is an infinitesimal magnetic dipole with density $\vec{M}$, is located at the point Q described by the spherical polar coordinates ($r=a$, $\theta=90°$, $\phi=0°$). Each point P on the surface of the cylinder is defined by a "geodetical polar coordinate" system (σ,β), where σ is the arclength of the geodesic connecting Q to P and β is the angle between $\hat{\phi}$ (at point Q) and geodesic QP. The local orthonormal basis vectors ($\hat{\sigma},\hat{\beta}$) are also associated with these two parameters. The observation point in the far field is specified by its spherical polar coordinates (r,θ,ϕ). The radiated field at an arbitrary point can be expressed in terms of two potentials, Φ and Ψ, which, in cylindrical coordinates, can be written as:

$$\Phi = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{-jn\phi} \int_{-\infty}^{\infty} f_n(k_z) \cdot H_n^{(2)}(k_t\rho)\, e^{-jk_z z}\, dk_z \tag{37}$$

$$\Psi = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{-jn\phi} \int_{-\infty}^{\infty} g_n(k_z) \cdot H_n^{(2)}(k_t\rho) \cdot e^{-jk_z z}\, dk_z \tag{38}$$

For the problem under consideration, we can express the spectral weight coefficients as

$$f_n(k_z) = \frac{j\omega\varepsilon M_\phi}{2\pi k_t^2 H_n^{(2)}(k_t a)} \tag{39}$$

$$g_n(k_z) = \frac{1}{k_t H_n^{(2)\prime}(k_t a)} \left[\frac{-M_z}{2\pi} + \frac{n k_z M_\phi}{2\pi k_t^2 a}\right] \tag{40}$$

where

$$k_t = \begin{cases} \sqrt{k^2 - k_z^2} & , \ k > k_z \\ -j\sqrt{k_z^2 - k^2} & , \ k < k_z \end{cases} \tag{41}$$

In order to derive an asymptotic expansion of (37) and (38), we proceed as follows. As a first step, we apply Watson's transformation to the infinite summation with respect to n and employ appropriate asymptotic formulas for Hankel functions with large order and argument to derive the following expressions for (37) and (38) under the conditions that ka is large and ϕ small compared to π:

$$\Phi \sim \frac{\omega \varepsilon M_\phi}{(2\pi)^2} \sqrt{\frac{2\pi}{\rho}} \ e^{j\pi/4} \int_{-\infty}^{\infty} dk_z \ e^{-j\Omega} \cdot \frac{m^2}{k^{5/2}} \cdot f_0 \ (\xi_1) \tag{42}$$

$$\Psi \sim \frac{jM_\phi}{(2\pi)^2} \sqrt{\frac{2\pi}{\rho}} \cdot \frac{e^{j\pi/4}}{2} \cdot \int_{-\infty}^{\infty} dk_z \cdot e^{-j\Omega} \cdot \frac{k_z}{k_t^{5/2}} [jm \, g_1(\xi_1) + 2m^3 g_0(\xi_1)]$$

$$-j\frac{M_z}{(2\pi)^2} \cdot a \sqrt{\frac{2\pi}{\rho}} \cdot \frac{e^{j\pi/4}}{2} \cdot \int_{-\infty}^{\infty} dk_z \cdot \frac{e^{-j\Omega}}{k_t^{1/2}} \cdot g_0(\xi_1) \tag{43}$$

where

$$\Omega = k_z z + k_t[\rho+a(\phi-\pi/2)]$$

$$m = (k_t a/2)^{1/3}$$

$$\xi_1 = m(\phi-\pi/2)$$

f_0, g_0, g_1 = Fock's functions defined in Appendix A.

M_ϕ and M_z = components of $\vec{M}$, $(\vec{M}\cdot\hat{n}=0)$

Next, applying the "saddle-point" technique to (42) and (43) and keeping only the first-order terms, the far field can be written in terms of its components along the normal and tangent to the surface at the "stationary point" P_1 as

$$E_{n_2} = (\vec{M}\cdot\hat{\beta}_1)\ \left(\frac{jke^{-jk\sigma}}{4\pi}\right)\cdot g_o(\xi_{1s})\cdot\frac{e^{-jkR}}{R}$$
$$+\frac{(\vec{M}\cdot\hat{\phi}_1)\ (\hat{\phi}_1\cdot\hat{\beta}_1)}{4\pi}\cdot e^{-jk\sigma}\cdot\left(\frac{k\rho_\sigma}{2}\right)^{1/3}\cdot g_1(\xi_{1s})\cdot\frac{e^{-jkR}}{R} \tag{44}$$

$$E_{\beta_2} = -\frac{(\vec{M}\cdot\hat{\phi}_1)\ (\hat{\phi}_1\cdot\hat{\sigma}_1)}{2\pi}\cdot\left(\frac{k\rho_\sigma}{2}\right)^{2/3}\cdot e^{-jk\sigma}\cdot f_0(\xi_{1s})\cdot\frac{e^{-jkR}}{R} \tag{45}$$

where

P_1: is the stationary point of Ω which turns out to be the same as the point of diffraction predicted by GTD.

$\xi_{1s} = (\frac{ka}{2})^{1/3} (\phi-\pi/2) \cdot \sin^{1/3} \theta$

ρ_σ = radius of curvature of geodesic QP_1

σ = arc length QP_1

R = the distance between the point of diffraction P_1 and the observation point

$n_2 = \hat{\sigma}_2 \times \hat{\beta}_2$; normal to the surface at P_1

The details of the derivations of (44) and (45) are given in Appendix B.

Fig. 12 illustrates the geometric meaning of some of the parameters appearing in (44) and (45), for the observation point is located in the shadow region. In this case, ξ_{1s}, which is identical to ξ given in (16), is the reduced distance traveled by the surface ray before leaving the surface tangentially.

In the lit region, the geometric interpretations of σ and ξ are shown in Fig. 13. The rays, like QP_1P, that do not obey the generalized Fermat's principle are called "psuedo-rays" [25]. The ray QP_1P appears to travel along the surface up to the point P_1 and then leaves the surface at P_1 tangentially in the opposite direction, to reach the observation point P. It should be noted that formulas (44) and (45) give us the contribution of the ray which

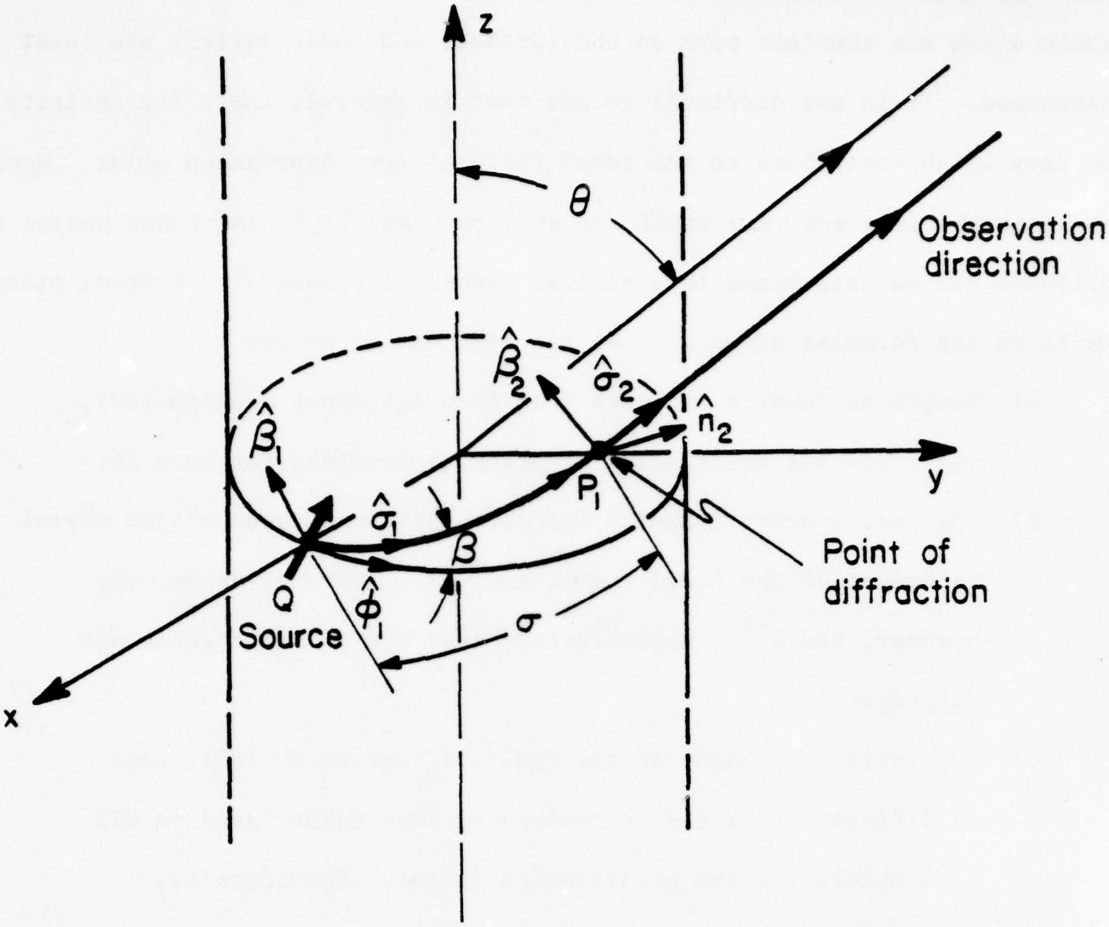


Figure 12: Diffraction of rays by a cylindrical body.

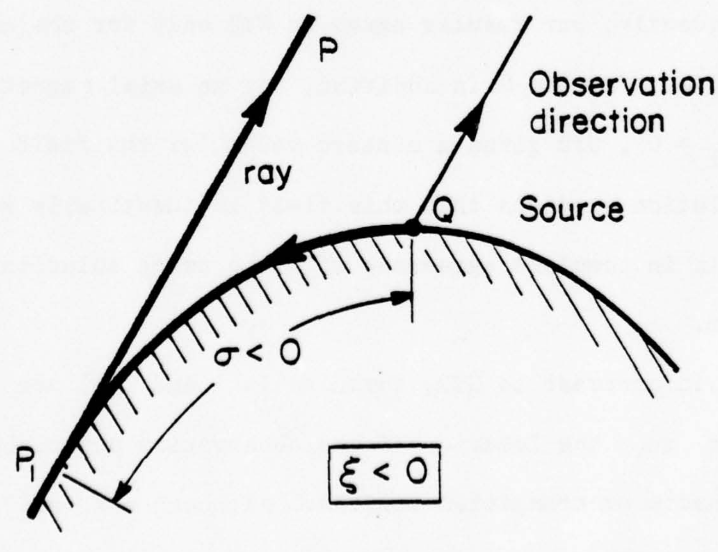


Figure 13: Diffraction of "psuedo-rays."

travels along the shortest path on the surface, and thus, suffers the least attenuation. It is not difficult to see that, in general, there are infinitely many rays which contribute to the total field at any observation point. However, their contributions are very small compared to that of QP_1 and their phases and amplitudes can be determined in a similar manner (Appendix B). Several other remarks on the formulas given in (44) and (45) are in order:

a) Numerical results indicate that good agreement between (49), (45), and the exact modal solution is obtained for ka > 10.

b) The zeroth-order terms in the asymptotic expansion of the normal component of the field $\bar{E}$ are identical to those given by GTD; however, the $k^{-1/3}$ terms derived from the two approaches are different.

c) Tangential component of the field, E_β, given by (45), also is different from the corresponding expression based on GTD by a multiplicative polarization factor. Specifically,

$$(45) = \left[\frac{(\vec{M}\cdot\hat{\phi}_1)}{(\cos\beta)\cdot(\vec{M}\cdot\hat{\sigma}_1)}\right] \text{(GTD)} \tag{46}$$

Consequently, our results agree in GTD only for the circumferential ray, i.e., for $\beta = 0$. In addition, for an axial magnetic dipole $(\vec{M}\cdot\hat{\phi}_1 = 0)$, GTD gives a nonzero value for the field in the $\hat{\beta}_2$ direction our solution predicts that this field is identically zero, a result which is in complete agreement with the exact solution for the problem.

In contrast to GTD, formulas (44) and (45) are valid in respect to the location of the observation point, be it in the lit, shadow or transition regions. Although not valid in the paraxial region ($\beta \approx 90°$), they can be generalized to work along this direction.

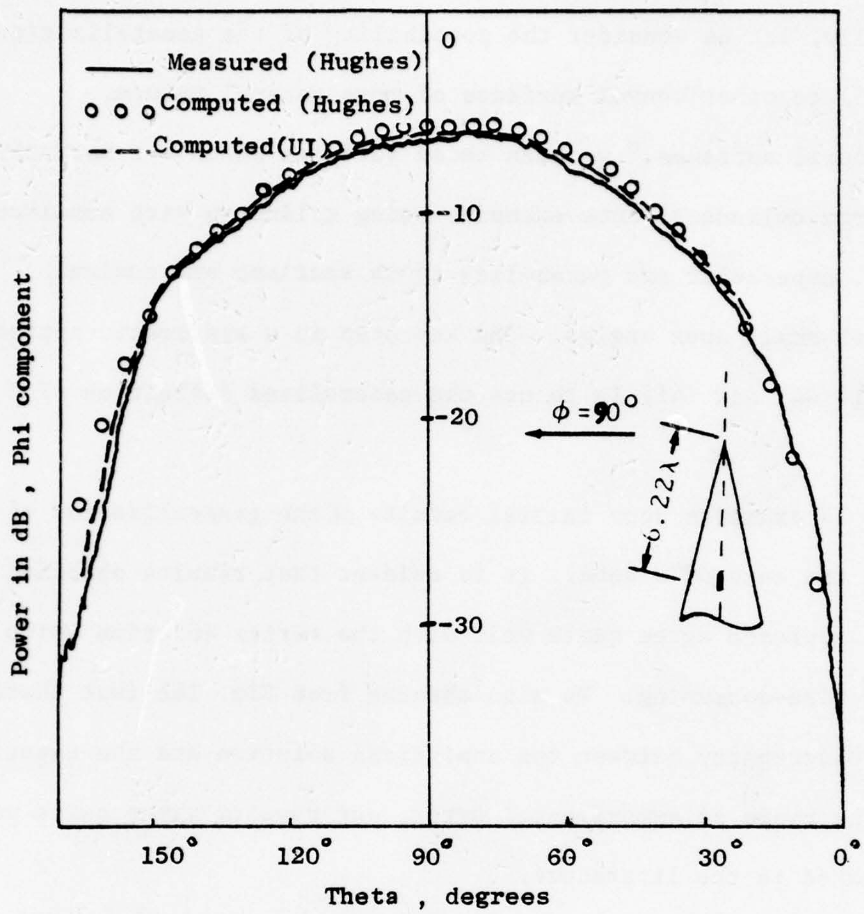
Finally, let us consider the possibility of the generalization of (44) and (45) to other convex surfaces of more general nature. By "more general surfaces," we mean those surfaces which are not substantially different from cylinders, some examples being cylinders with noncircular (elliptical, hyperbolic and parabolic) cross sections and conical surfaces with small apex angles. The key step in a systematic approach to generalizing (44) and (45) is to use the generalized definition of ξ given in (16).

Fig. 14 exhibits some initial results of the generalization of these formulas to the case of a cone. It is evident that results obtained from the present approach agree quite well with the series solution which is rather tedious and time-consuming. We also observe from Fig. 14c that there is a noticeable discrepancy between the analytical solution and the experiment. Thus, within the range of experimental error, our results agree quite well with those published in the literature.

3.3 Approach Based on an Asymptotic Evaluation of the Radiation Integral of the Surface Current

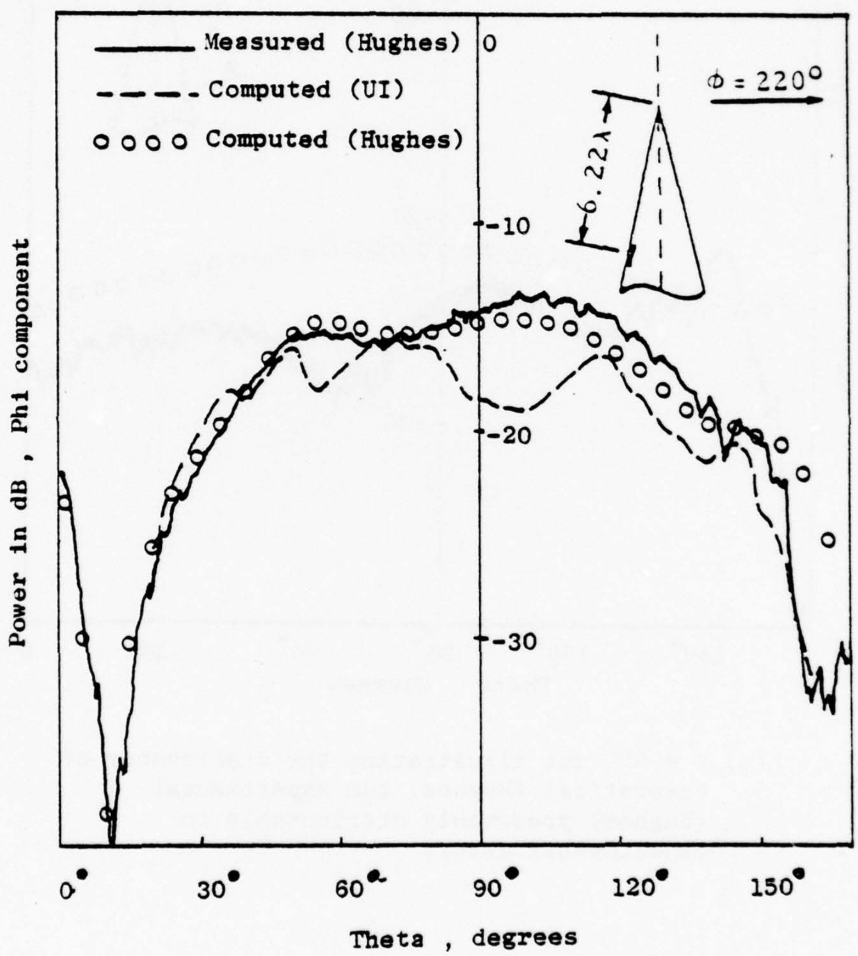
As a final topic, we consider an approach based on the asymptotic evaluation of the radiation integral expressed in terms of the induced surface current which is itself derived in an asymptotic manner for surfaces with large radius of curvature.

It was shown in Sec. 2 that Fock's theory can provide us with an expression for the scattered field in the neighborhood of a smooth convex body illuminated by a plane wave. Using this solution in conjunction with the reciprocity principle, we can find the far field radiated by a point source located on the surface of the body. By generalizing the definition of ξ in

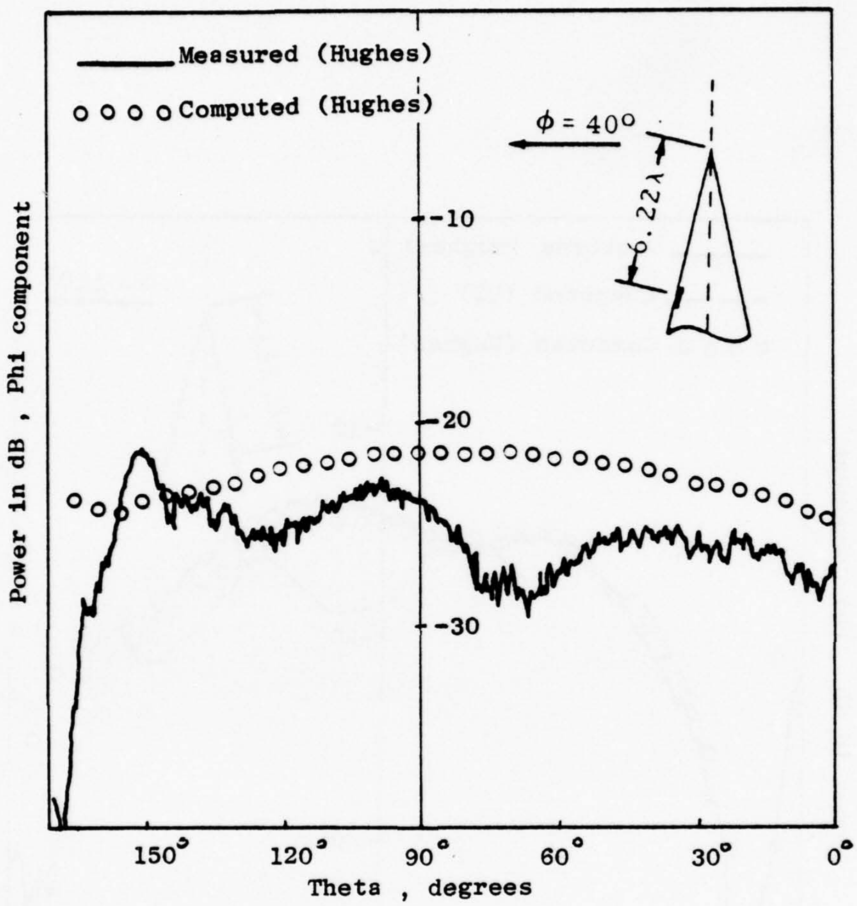


(a) ϕ = 90° cut;

Figure 14: Comparison between spectral domain results (UI), modal approach (Hughes), and experimental measurements (Hughes). The UI results are derived from a generalized version of (44) and (45) for a cone. The Hughes' results have been reproduced from [63] and are based on a modal series of 13 terms. All results are for $\lambda/2$ radial slot on a cone of half-angle 10°.


Measured (Hughes)
Computed (UI)
Computed (Hughes)
φ = 220°
6.22λ
0
-10
-20
-30
Power in dB , Phi component
0°
30°
60°
90°
120°
150°
Theta , degrees

(b) ϕ = 220° cut



(c) ϕ = 40° cut illustrating the discrepancy of theoretical (Hughes) and experimental (Hughes) presumably attributable to experimental error.

Fock's theory, we can also write the final result in a GTD format and represent it as a surface ray. The total field at a point on the surface is obtained by adding all the possible rays which reach the observation point P. Various techniques can be used to determine the field propagation along these rays. For instance, when the source is located on the surface, and the surface is a conical one, the field at each point can be decomposed into two parts.

$$F = F_1 + F_2 \tag{47}$$

where F_1 is the geometrical optics field when the observation point is directly illuminated by the source, and is the creeping-wave contribution derivable via an extension of Fock's theory when the point is in the shadow region. The other term, F_2, is the so-called tip contribution, and can be obtained by physical optics or GTD. Goodrich et al. [42] have applied this procedure to find the radiation pattern of slot arrays on cones.

The approximate induced surface current distribution can be obtained by Fock's theory, GTD [13], [14], [16] and [25] or some other appropriate high frequency technique. The induced surface current due to a magnetic dipole on a perfectly conducting circular cylinder and cone has been calculated by Chang et al. [44], and Chan et. al [45] whose procedure is based upon an asymptotic expansion of the exact modal solution to the above-mentioned problems. Lee, et al. [46] and [22] have treated the same problem by a method based on Fock's asymptotic solution of the problem of a sphere [47]. These expressions for the current distribution can be used in the radiation integral representation of the far field.

The numerical evaluation of this integral is a formidable task, especially when the frequency is very high. Thus, it is highly desirable to have an analytical and explicit formula for the far field expressed in terms of the surface current. We now discuss an approach for accomplishing this task and examine the problem of deriving an asymptotic expansion of the far field radiated due to a point source located on the surface of a smooth, conducting, and convex body of an arbitrary shape.

Consider an arbitrary smooth convex surface S shown in Fig. 15. Let a magnetic dipole source be located at a point Q on S. We parametrize the surface S introducing a "geodetical polar coordinate" system with the pole located at Q such that an arbitrary point P_1 on the surface is defined by a pair of numbers (σ,β), where is the arc length of the geodesic QP_1 and is the angle between QP_1 and some reference direction at Q. Unit vectors along the constant parameter curves $\hat{\sigma}$ and $\hat{\beta}$ are locally orthogonal. The unit normal to the surface, $\hat{n}$, is given by $\hat{n} = \hat{\sigma} \times \hat{\beta}$. An element of length in this coordinate system may be written as

$$ds^2 = d\sigma^2 + G\, d\beta^2 \tag{48}$$

The radiation integral for the scattered far field can be written

$$\vec{E} = \frac{-j\omega\mu}{4\pi} \iint_S \vec{J}(1-\hat{R}\hat{R}) \cdot \frac{\exp(-jkR)}{R}\, dS \tag{49}$$

where R is the distance between any point on the surface and the observation point. In the geodetical polar coordinate system, we can rewrite a scalar component of (49), say M, in terms of a double integral of the following general form

$$M = \iint_D F(\sigma,\beta,P)\, \frac{\exp[-jk(R+\sigma)]}{R}\, \sqrt{G}\, d\sigma\, d\beta \tag{50}$$

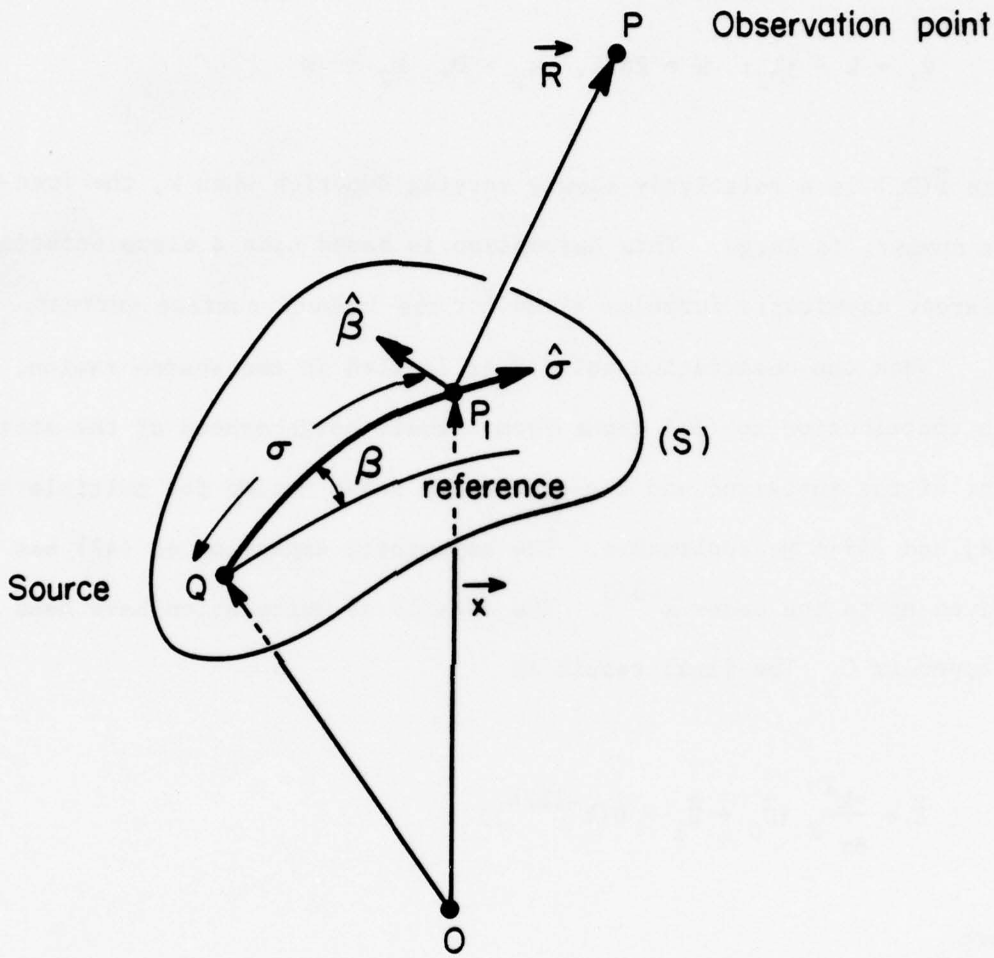


Figure 15: Source radiation in the presence of a smooth convex surface, parametrized by geodetical polar coordinate system.

where we have assumed the following form for the surface current:

$$\vec{J}(P_1) = \vec{f}(P_1)\exp(-jk_1\sigma) = J_\sigma\hat{\sigma} + J_\beta\hat{\beta}$$

$$k_1 = k - jk_2; \quad k = 2\pi/\lambda, \quad k_2 > 0, \quad k_2 << k \tag{51}$$

where $\vec{f}(P_1)$ is a relatively slowly varying function when k, the free-space wave number, is large. This assumption is based upon a close scrutiny of different asymptotic formulas given for the induced surface current.

When the observation point P is located in the shadow region, the main contribution to (51) comes from a small neighborhood of the stationary point of the integrand, and the stationary phase method for multiple integrals ([48] and [49]) is applicable. The asymptotic expansion of (49) has been derived up to the order $k^{-5/3}$. The details of calculation have been presented in Appendix C. The final result is

$$\vec{E} = \frac{-k^2}{8\pi^2}\left(\vec{U}_0 + \vec{U}_1 + 0(k^{-11/6})\right) \tag{52}$$

where

$$U_0 = \hat{\beta}\, J_\beta \cdot \frac{D_0}{k^{5/6}}\sqrt{\frac{\rho_g}{R(R+\rho_g)}} \cdot e^{-jkR} \tag{53}$$

$$U_1 = \left[(AJ_\beta + \frac{\partial}{\partial\sigma}(J_\beta e^{jk\sigma})e^{-jk\sigma})\hat{\beta} + (BJ_\beta + CJ_\sigma)\hat{n}\right] \cdot \frac{D_1}{k^{7/6}}\sqrt{\frac{\rho_g}{(R+\rho_g)R}} \cdot e^{-jkR} \tag{54}$$

$$D_0 = e^{-j\pi/4} \cdot 6^{5/6} \cdot \Gamma(1/2) \cdot \Gamma(1/3) \cdot \rho_\sigma^{2/3} \tag{55}$$

$$D_1 = \frac{-je^{-j\pi/4}}{3} \cdot 6^{7/6} \cdot \Gamma(2/3) \cdot \Gamma(1/2) \cdot \rho_\sigma^{4/3} \tag{56}$$

A, B, and C are dependent upon geometric properties of the surface at the stationary point which turns out to be exactly the same as the "point of diffraction" of surface rays. The quantities A, B, and C are given by

$$\begin{aligned} A = \frac{1}{2G} \frac{\partial}{\partial\sigma}G - \frac{\rho_\sigma}{2} \cdot \frac{\partial}{\partial\sigma} \left(\frac{1}{\rho_\sigma}\right) + \frac{\rho_g}{2G} \Big[\frac{L^{\beta\beta}}{\rho_\sigma} \\ + (L^{\beta\sigma})^2 - (1/2)\frac{\partial^2 G}{\partial\sigma^2}\Big] + 0 \left(\frac{1}{R}\right) \end{aligned} \tag{57}$$

$$B = L^{\beta\sigma}/G^{1/2}, \qquad C = -1/\rho_\sigma \tag{58}$$

where

ρ_σ = radius of curvature of the geodesic

ρ_g = geodetic radius of curvature

$L^{\beta\beta}$, $L^{\beta\sigma}$ = coefficients of the second fundamental form of the surface (S)

A geometric interpretation of these parameters has been illustrated in Fig. 16. It is evident from this figure that $\left[\frac{\rho_g}{R(R+\rho_g)}\right]^{1/2}$ is simply the divergence factor of the rays leaving the surface tangentially at the point of diffraction. In using formula (56), we should bear in mind that the various terms in U_0 and U_1 are not of the same order. For example, in the deep shadow, J_σ is exponentially larger than J_β.

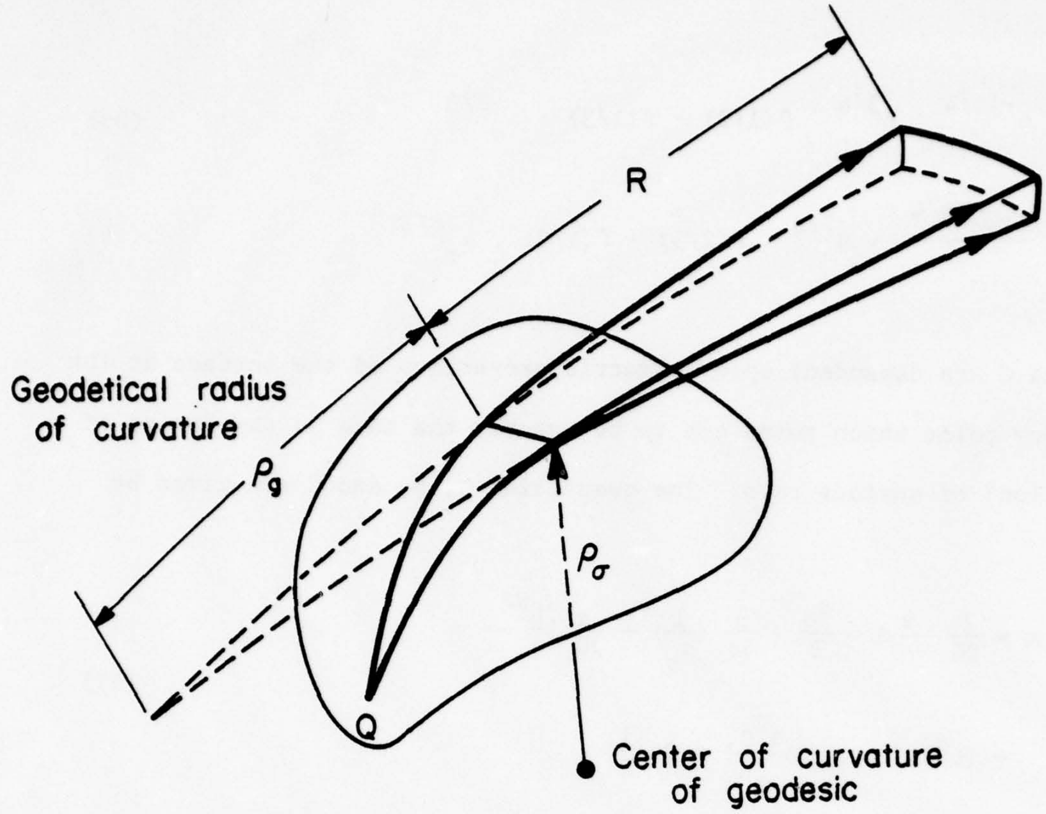


Figure 16: Diffraction of rays by a smooth convex body and geometric meaning of quantities ρ_g, ρ_σ and R.

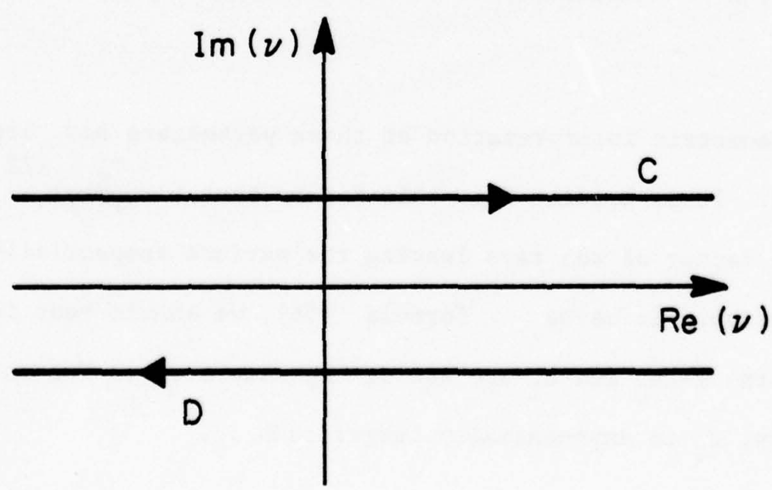


Figure 17: Paths C and D in Watson transformation.

The formulas given in (56) have been tested and compared with other available solutions. An important conclusion derived from this comparison is that although the method of radiation integral is based on less restrictive assumptions, it is perhaps not as useful as the spectral domain approach because the stationary point of the phase of the integrand in (50) is of the second order, and hence, the asymptotic expansion of this integral converges rather slowly except when $k\rho_\sigma$ is very large ($\approx$40 or more).

ACKNOWLEDGEMENT

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APPENDIX A: FOCK FUNCTIONS

In studies of radio-wave propagation around the earth by Van der Pol, Bremmer, Pryce, Fock, and others, and also the later studies of diffraction of electromagnetic waves by certain bodies of revolution ([51],[52],[53],[54],[55],[48],[56],[57],[58],[59],[60] and [50]), a class of universal functions was introduced which can be used to predict the amplitude and the phase of the reflected or diffracted field by smooth convex surfaces [17]. An exhaustive treatment of these functions which, in general, are defined as Fourier integrals having combinations of Airy integrals in their integrands, has been carried out by N. A. Logan [61]. (See also Bowman, et al. [1] and Logan and Yee [17]).

Since the first extensive application of these functions to diffraction theory was done by Fock, many authors named them after him. Here we list only the most important formulas and expressions for these functions without going through the details of their derivations. We have followed Logan's set of notations for these functions [61]. However, since his time dependence factor, $\exp(-i\omega t)$, is different from one we have used throughout this paper, namely $\exp(+j\omega t)$, our expressions, listed below, are conjugates of what have been presented in [61].

First we start with general definitions. Fock's most general form of the "Van der Pol-Bremmer diffraction formula" is

$$V(x,y_1,y_2,q) = \exp(j\pi/4) \cdot \sqrt{\frac{x}{\pi}} \cdot \int_{-\infty}^{\infty} \cdot e^{-jxt} \cdot w_2(t - y_>) \cdot \left\{ v(t - y_<) - \frac{v'(t) - qv(t)}{w_2'(t) - qw_2(t)} \cdot w_2(t - y_>) \right\} dt \qquad \text{(A.1)}$$

where $w_1(t)$, $w_2(t)$, $u(t)$ and $v(t)$ are Fock-type Airy functions, defined as

$$u(t) = \sqrt{\pi}\,\mathrm{Bi}(t) \qquad , \qquad v(t) = \sqrt{\pi}\,\mathrm{Ai}(t)$$

$$w_1(t) = u(t) + jv(t) \qquad , \qquad w_2(t) = w_1(t)^*$$

We note that w_1 and w_2 can also be defined as in Sec. 2. $y_>$ and $y_<$ are the larger and smaller of the two numbers y_1 and y_2. $V(x,y_1,y_2,q)$ is proportional to the attenuation suffered by an electromagnetic wave generated by a source located at reduced height y_1 above the surface of a smooth convex body, when it reaches the observation point located at reduced height y_2 above the same surface. x is the reduced distance between the source and the observation point along the surface, and q is dependent upon the impedance of the surface. Let us consider some useful limiting cases.

When $y_1 = y_2 = 0$, then $V(x,0,0,q)$ is denoted by V_0, where

$$V_0(x,q) = \frac{e^{j\pi/4}}{2} \cdot \sqrt{\frac{x}{\pi}} \cdot \int_{-\infty}^{\infty} \frac{e^{-jxt}\, w_2(t)}{w_2'(t) - qw_2(t)} \cdot dt \tag{A.2}$$

We also have

$$v(x) = V_0(x,0) = \frac{e^{j\pi/4}}{2} \sqrt{\frac{x}{\pi}} \int_{-\infty}^{\infty} \frac{e^{-jxt}\, w_2(t)}{w_2'(t)}\, dt \tag{A.3}$$

$$u(x) = \lim_{q\to\infty} \left[-2jxq^2 V_0(x,q)\right] = \frac{e^{j3\pi/4}}{\sqrt{\pi}} \cdot x^{3/2} \int_{-\infty}^{\infty} \frac{e^{-jxt}\, w_2'(t)}{w_2(t)} \tag{A.4}$$

When $y_1 = 0$ and $y_2 \to \infty$, then $V \to V_1(x,q)$:

$$V_1(x,q) = \frac{1}{\sqrt{\pi}} \cdot \int_{-\infty}^{\infty} \frac{e^{-jxt}}{w_2'(t) - qw_2(t)} \cdot dt \tag{A.5}$$

and also

$$g(x) = V_1(x,0) = \frac{1}{\sqrt{\pi}} \cdot \int_{-\infty}^{\infty} \frac{e^{-jxt}}{w_2'(t)}\, dt \tag{A.6}$$

$$f(x) = \lim_{q \to \infty} \left[- qV_1(x,q) \right] = \frac{1}{\sqrt{\pi}} \cdot \int_{-\infty}^{\infty} \frac{e^{-jxt}}{w_2(t)} \, dt \tag{A.7}$$

Based on Equations (A.6) and (A.7), a class of functions can be defined:

$$f^{(n)}(x) = \frac{(-j)^n}{\sqrt{\pi}} \cdot \int_\Gamma \frac{t^n \cdot e^{-jxt}}{w_2(t)} \cdot dt = \frac{d^n f(x)}{dx^n} \tag{A.8}$$

$$g^{(n)}(x) = \frac{(-j)^n}{\sqrt{\pi}} \cdot \int_\Gamma \frac{t^n \quad e^{-jxt}}{w_2'(t)} \cdot dt = \frac{d^n g(x)}{dx^n} \tag{A.9}$$

where Γ is any path in the complex t-plane which comes from $-\infty$ in a sector defined by $-\pi \leq \arg(t) < -\frac{\pi}{3}$ and goes to $+\infty$ in the sector $-\frac{\pi}{3} < \arg(t) < \frac{\pi}{3}$. In what follows, we will give the suitable formulas for $f(x)$ and $g(x)$ in different ranges. Tabulated values and graphs of these functions can be found in [57],[54] and [61].

When x is very large and negative, the following asymptotic expansions for $f(x)$ and $g(x)$ can be used [61]:

$$f(x) \sim -2jxe^{jx^3/3} \left\{ 1 + \frac{j}{4x^3} + \frac{1}{2x^6} - \frac{j175}{64x^9} - \frac{395}{16x^{12}} + \frac{j318175}{1024x^{15}} + \ldots \right\} \tag{A.10}$$

$$g(x) \sim 2e^{jx^3/3} \left\{ 1 - \frac{j}{4x^3} - \frac{1}{x^6} + \frac{j469}{64x^9} + \frac{5005}{64x^{12}} - \frac{j1122121}{1024x^{15}} - \ldots \right\} \tag{A.11}$$

The above formulas are valid and accurate for $x \ll -1$. For moderate values of x, namely, $-1 \leq x \leq 1$, it is difficult to find an appropriate expression. Although there are some analytical techniques like "stationary phase method" or "Poisson summation formula" which may be used to evaluate $f^{(n)}$ and $g^{(n)}$ for these values, another possible way which is probably easier and more efficient is to interpolate the tabulated values of these functions in this range.

In the vicinity of zero ($|x| \approx 0$), the Taylor expansion can be used to calculate f and g. The coefficients are given by

$$f^{(n)}(0) = e^{-j(5n\pi/6-\pi/3)} \cdot \sqrt{\pi} \cdot \left(\frac{3\pi}{2}\right)^{(2/3)(n-1/4)} \cdot \sum_{m=0}^{\infty} A_m(n) \cdot \left(\frac{2}{3\pi}\right)^{2m}$$

$$\cdot \, \tau\left(2m - \frac{4n-1}{6} \, , \, \frac{3}{4}\right) \tag{A.12}$$

$$g^{(n)}(0) = e^{-j5n\pi/6} \cdot \sqrt{\pi} \cdot \left(\frac{3\pi}{2}\right)^{(2/3)(n-3/4)} \cdot \sum_{m=0}^{\infty} B_m(n) \cdot \left(\frac{2}{3\pi}\right)^{2m}$$

$$\cdot \, \tau\left(2m - \frac{4n-3}{6} \, , \, \frac{1}{4}\right) \tag{A.13}$$

where $\tau(\lambda,\mu)$ is the generalized "tau" function:

$$\tau(\lambda,\mu) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+\mu)^{\lambda}} \quad , \quad \lambda > 1 \tag{A.14}$$

$$A_0(n) = 1 \quad , \quad A_1(n) = \frac{5}{48}(n-1)$$

$$A_2(n) = 5\left(5n^2 - 143n + \frac{26385}{16}\right)/(2^9 \cdot 3^2)$$

$$B_0(n) = 1 \quad , \quad B_1(n) = -7(n - 3/2)/48$$

$$B_2(n) = (49n^2 + 364n + 39849/16)/(2^9 \cdot 3^2)$$

When x is large, and positive, residue series can be used to compute $f^{(n)}$ and $g^{(n)}$:

$$f^{(n)}(x) = e^{j(2+7n)\pi/6} \sum_{p=1}^{\infty} \frac{(r_p)^n \exp(r_p \cdot x \cdot e^{-j5\pi/6})}{Ai'(-r_p)} \tag{A.15}$$

$$g^{(n)}(x) = e^{j7\pi n/6} \sum_{p=1}^{\infty} \frac{(r_p')^{n-1} \exp(r_p' \cdot x \cdot e^{-j5\pi/6})}{Ai(-r_p')} \tag{A.16}$$

where $Ai(-r_p) = 0$ and $Ai'(-r_p') = 0$ for $p = 1,2,3,\ldots$.

APPENDIX B: DERIVATION OF FORMULAS (44) and (45)

Here, we consider only the derivation of the asymptotic expansion of Φ for a circumferential magnetic dipole. In this case, Φ may be written as:

$$\Phi = \frac{-j\omega\varepsilon M_\phi}{(2\pi)^2} \int_{-\infty}^{\infty} dk_z \cdot \frac{e^{-jk_z z} \cdot S(k_t)}{k_t^2} \tag{B.1}$$

where

$$S(k_t) = \sum_{n=-\infty}^{\infty} e^{-jn\phi} \cdot \frac{H_n^{(2)}(k_t\rho)}{H_n^{(2)}(k_t a)} \tag{B.2}$$

Applying the Watson transformation to (B.2),

$$S(k_t) = \frac{j}{2} \cdot \int_{C+D} \frac{H_\nu^{(2)}(k_t\rho)}{H_\nu^{(2)}(k_t a)} \cdot \frac{e^{-j\nu(\phi-\pi)}}{\text{Sin } \nu\pi} \cdot d\nu \tag{B.3}$$

where C and D are shown in Figure 17. Or,

$$S(k_t) = j \cdot \int_{-\infty}^{\infty} \frac{\cos \nu(\pi - \phi) \quad H_\nu^{(2)}(k_t\rho)}{\sin \nu\pi \cdot H_\nu^{(2)}(k_t a)} \cdot d\nu \tag{B.4}$$

Substituting the expansion

$$\frac{\cos \nu(\pi - \phi)}{\sin \nu\pi} = j \sum_{i=1}^{2} \sum_{l=0}^{\infty} e^{-j\nu(\phi_i + 2\pi l)} \tag{B.5}$$

where $\phi_1 = \phi$ and $\phi_2 = 2\pi - \phi$, in (B.4), the result will be:

$$S(k_t) = -\sum_{i=1}^{2} \sum_{l=0}^{\infty} \int_{-\infty}^{\infty} \frac{H_\nu^{(2)}(k_t\rho)}{H_\nu^{(2)}(k_t a)} \cdot e^{-j\nu(\phi_i + 2\pi l)} \cdot d\nu \tag{B.6}$$

Each term of the above expansion is associated with a "creeping wave" travelling in a counterclockwise (i = 1) or clockwise (i = 2) direction around the cylinder. Following the ray concept, each creeping wave appears to be travelling along a specific surface ray. Now, as $\rho \to \infty$ (far zone) for each fixed ν, we have [62]

$$H_\nu^{(2)}(k_t\rho) \sim \sqrt{\frac{2}{\pi k_t \rho}} \cdot e^{-j(k_t\rho - \nu\pi/2 - \pi/4)} \tag{B.7}$$

On the other hand, it can be shown that the significant contribution to $S(k_t)$ comes from a small neighborhood of $k_t a$. In this neighborhood, where $k_t a$ and ν are large and close to each other ($|k_t a - \nu| \lesssim |\nu|^{1/3}$), the Hankel's asymptotic expansion (B.7) is not valid any longer. In this case, it is necessary to expand Bessel's functions in terms of Fock-type, Airy functions, $w_1(t)$ and $w_2(t)$, and their derivatives [16]:

$$H_\nu^{(2)}(x) \sim \frac{j}{m\sqrt{\pi}} \left\{ w_2(t) - \frac{1}{60m^2} \left[4t\, w_2(t) + t^2\, w_2'(t) \right] + \ldots \right\} \tag{B.8}$$

$$H^{(2)'}(x) \sim \frac{-j}{m^2\sqrt{\pi}} \left\{ w_2'(t) + \frac{1}{60m^2} \left[4t\, w_2'(t) + (6 - t^3)\, w_2(t) \right] + \ldots \right\} \tag{B.9}$$

where

$$m = \left(\frac{x}{2}\right)^{1/3} \quad , \quad t = \frac{\nu - x}{m} \quad \text{(m is very large)}$$

Inserting (B.7) and the first-order terms of (B.8) and (B.9) into (B.6) and (B.1), we obtain

$$\Phi \sim \frac{\omega \varepsilon M_\phi}{(2\pi)^2} \cdot \sqrt{\frac{2\pi}{\rho}} \cdot e^{j\pi/4} \cdot \sum_{i=1}^{2} \sum_{l=0}^{\infty} \int_{-\infty}^{\infty} dk_z \cdot e^{-j\Omega_{il}} \cdot f_0(\xi_{il}) \cdot \frac{m^2}{k_t^{5/2}} \tag{B.10}$$

where

$$m = (k_t a/2)^{1/3}$$

$$\Omega_{il} = k_z z + k_t[\rho + a(\phi_i + 2\pi l - \pi/2)]$$

$$\xi_{il} = m(\phi_i + 2\pi l - \pi/2)$$

Introducing a new integration variable α:

$$k_z = k \sin \alpha \tag{B.11}$$

$$k_t = k \cos \alpha \tag{B.12}$$

$$\beta_{il} = \tan^{-1}\{z/[\rho + a(\phi_i + 2\pi l - \pi/2)]\} \tag{B.13}$$

we have:

$$\Omega_{il} = kR_{il} \cos(\beta_{il} - \alpha) \tag{B.14}$$

where

$$R_{il} = \left\{z^2 + [\rho + a(\phi_i + 2\pi l - \pi/2)]\right\}^{1/2} .$$

Now (B.10) takes the following form:

$$\Phi \sim \frac{\omega\varepsilon M_\phi}{(2\pi)^2} \cdot \sqrt{\frac{2\pi}{\rho}} \cdot \frac{m_0^2}{k^{3/2}} \cdot e^{j\pi/4} \cdot \sum_{i=1}^{2} \sum_{l=0}^{\infty} \int_\gamma d\alpha \cdot e^{-jkR_{il}\cos(\alpha-\beta_{il})}$$

$$\cdot \cos^{-5/6} \alpha \cdot f_0(\xi_{il}) \tag{B.15}$$

γ is the path of integration in the complex α-plane, which is shown in Figure 18.

Now we deform the path of integration into the "steepest descent path," SDP, passing through the saddle point of the phase of the integrand. Performing the "saddle point integration," we can derive the asymptotic expansion of (B.15) for large kR_{il}. The first order term is:

$$\Phi \sim \frac{\omega\varepsilon M_\phi}{2\pi k^2} \cdot e^{j\pi/2} \cdot \left(\frac{ka}{2}\right)^{1/3} \cdot \sum_{i=1}^{2} \sum_{l=0}^{\infty} (\cos \beta_{il})^{-4/3} \cdot \frac{e^{-jkR_{ils}}}{R_{ils}} \cdot f_0(\xi_{ils}) \tag{B.16}$$

where R_{ils} and ξ_{ils} are the values of these parameters at the stationary point specified by $\alpha = \beta_{il}$.

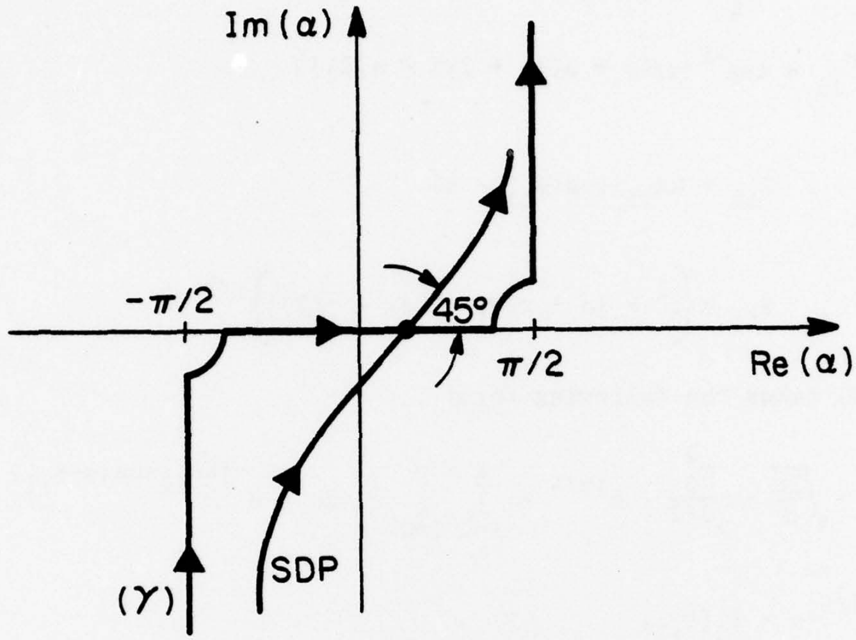

Figure 18: Steepest descent path (SDP) for integral (B.15).

Eqn. (B.16) is the creeping-wave representation of the far field. If the cylinder is large ($ka >> 1$) and $|\phi|$ is not very close to π, then only the first term ($i = 0$, $1 = 1$) has the most important contribution to the total infinite sum, and the other terms are not significant. Neglecting the other terms, we obtain the result given in (44 and 45). It should be emphasized that (44) and (45) are not valid when $|\beta|$ is close to $\pi/2$ (paraxial region), because in this case, $k_t a$ is very small, and (B.7), (B.8) and (3.9) no longer apply.

The other formulas can be derived in a similar manner.

APPENDIX C: ASYMPTOTIC EVALUATION OF THE RADIATION INTEGRAL

Consider the following double integral:

$$U(k) = \iint_D g(x,y) \cdot e^{jk\phi(x,y)}\, dxdy \tag{C.1}$$

where $g(x,y)$ is rather slowly varying, and $\phi(x,y)$ has a stationary point (x_s,y_s) inside domain D. The objective is to derive an asymptotic expansion for (C.1) when k is large.

Suppose g and ϕ have the following forms around (x_s,y_s):

$$\begin{cases} g(x,y) = (x - x_s)^{\lambda_0 - 1} (y - y_s)^{\mu_0 - 1} g_1(x,y), \quad \lambda_0, \mu_0, > 1 \\ \phi(x,y) = \phi(x_s,y_s) + a_{\delta,0}(x - x_s)^{\delta}[1 + P(x,y)] + b_{0,\tau}(y - y_s)^{\tau}[1 + Q(x,y)] \end{cases} \tag{C.2}$$

N. Chako [48] has derived the following asymptotic series for U:

$$U(k) \sim B_0 \cdot \sum_{p,q=0}^{\infty} A_{pq}(\alpha_1 + \alpha_2)(\beta_1 + \beta_2) \cdot \Gamma\left(\frac{\lambda_0 + p}{\delta}\right) \cdot \Gamma\left(\frac{\mu_0 + q}{\tau}\right)$$

$$\frac{1}{(ka_{\delta 0})^{\lambda_0/\delta}} \cdot \frac{1}{(kb_{0,\tau})^{q/\tau}} \tag{C.3}$$

where

$$B_0 = \frac{1}{(ka_{\delta,0})^{\lambda_0/\delta}} \cdot \frac{1}{(kb_{0,\tau})^{\mu_0/\tau}} \cdot \frac{1}{(\delta\tau)} \cdot e^{jk\phi(x_s,y_s)}$$

$$\alpha_1 = \exp[j\pi(\lambda_0 + p)/(2\delta)] \, , \; \alpha_2 = \exp\left\{[j\pi/(2\delta)]\left[(\lambda_0 + \rho)(2\delta + e^{j\pi\delta}) - 2\delta\right]\right\}$$

$$\beta_1 = \exp[j\pi(\mu_0 + q)/(2\tau)] \, , \; \beta_2 = \exp\left\{[j\pi/(2\tau)]\left[(\mu_0 + q)(2\tau + e^{j\pi\tau}) - 2\tau\right]\right\}$$

$$g_1(x,y) = \sum_{k,l=0}^{\infty} g_{k,l}(x - x_s)^k (y - y_s)^l$$

$$P(x,y) = \sum_{m+n\geq 1} a_{mn}(x - x_s)^m (y - y_s)^n$$

$$Q(x,y) = \sum_{m+n\geq 1} b_{mn}(x - x_s)^m (y - y_s)^n$$

$$A_{00} = g_{00} \quad , \quad A_{10} = g_{10} - g_{00}\left[(\lambda_0 + 1)\,\frac{a_{10}}{\delta} + \frac{b_{10}}{\tau}\right]$$

$$A_{01} = g_{01} - g_{00}\left[\frac{a_{01}}{\delta} + (\mu_0 + 1)\cdot\frac{b_{01}}{\tau}\right]$$

In order to apply this procedure to the integrals of the type (3.4) for which

$$\phi(x,y) = -\Omega(\sigma,\beta) = -(R + \sigma) \tag{C.4}$$

$$g(x,y) = F(\sigma,\beta,P)\,\frac{\sqrt{G}}{R} \tag{C.5}$$

When F is one of the components of $\vec{J}(1 - \hat{R}\hat{R})$, one should first determine the stationary point of Ω, wherein its first-order derivatives vanish. The second step is to compute the various order derivatives of Ω, J, $\hat{R}$, ..., at this point, and then insert them into (C.3). We just give the main formulas needed for these derivations.

Suppose the surface of the body, $\vec{x}(\sigma,\beta)$, is parametrized by a geodetical polar coordinate system. As discussed previously, in this system, σ is the arc length of the surface geodesic connecting the pole Q to $\vec{x}(\sigma,\beta)$, and β is the angle between the geodesic and some fixed reference geodesic at Q (Fig. 15).

The element of length in this system is given by

$$ds^2 = d\sigma^2 + G(\sigma,\beta)\,d\beta^2 \tag{C.6}$$

Let us denote $d\vec{x}(u)/du$ by $\vec{x}_u$; then we have the following set of relations

$$\vec{x}_{\beta\beta} = \frac{-\partial G/\partial\sigma}{2} \cdot \vec{X}_\sigma + \frac{\partial G/\partial\rho}{2G} \vec{x}_\beta + L^{\beta\beta} \vec{x}_3 \tag{C.7}$$

$$\vec{x}_{\beta\sigma} = \vec{x}_{\sigma\beta} = \frac{\partial G/\partial\sigma}{2G} \vec{x}_\beta + L^{\beta\sigma} \vec{x}_3 \tag{C.8}$$

$$\vec{x}_\sigma = \frac{-\vec{x}_3}{\rho_\sigma} \tag{C.9}$$

where $\vec{x}_\sigma = \hat{\sigma}$, and $\vec{x}_\beta/\sqrt{G} = \hat{\beta}$ are unit vectors along β = const. and σ = const. curves, and

$$\vec{x}_3 = \hat{n} = \frac{\vec{x}_\sigma \times \vec{x}_\beta}{\sqrt{G}} \tag{C.10}$$

is the outward unit normal to the surface. Another quantity of interest is the "geodetical curvature" κ_g given by

$$\kappa_g = \frac{\partial G/\partial\sigma}{2G} \tag{C.11}$$

Using the above relations, we can derive the following expressions which holds true at the stationary point:

$$\frac{\partial\Omega}{\partial\sigma} = 1 - \hat{R} \cdot \vec{X}_\sigma = 0 \tag{C.12}$$

$$\frac{\partial\Omega}{\partial\beta} = -\hat{R} \cdot \vec{x}_\beta = 0 \tag{C.13}$$

$$\frac{\partial^2\Omega}{\partial\sigma^2} = 0 \quad , \quad \frac{\partial^2\Omega}{\partial\sigma\partial\beta} = 0 \quad , \quad \frac{\partial^2\Omega}{\partial\beta^2} = G\left(\frac{1}{R} + \frac{1}{\rho_g}\right) \tag{C.14}$$

where $\rho_g = 1/\kappa_g$, and

$$\frac{\partial^3\Omega}{\partial\sigma^3} = \frac{1}{\rho_\sigma^2} \quad , \quad \frac{\partial^3\Omega}{\partial\sigma^2\partial\beta} = -L^{\beta\sigma}/\rho_\sigma \tag{C.15}$$

$$\frac{\partial^3\Omega}{\partial\beta^2\partial\sigma} = \frac{G}{R^2} + \frac{\partial G/\partial\sigma}{R} + \frac{\partial^2 G/\partial\sigma^2}{2} - \frac{L^{\beta\beta}}{\rho_\sigma} \tag{C.16}$$

$$\frac{\partial^3\Omega}{\partial\beta^3} = \frac{3\partial G/\partial\beta}{2R} + \frac{\partial G}{\partial\beta}\cdot\frac{\partial G}{\partial\sigma}\cdot\frac{1}{4G} + L^{\beta\sigma}\cdot L^{\beta\beta} + \frac{1}{2}\cdot\frac{\partial^2 G}{\partial\sigma\partial\beta} \tag{C.17}$$

where ρ_σ is the radius of curvature of the geodesic.

Equations (C.12) and (C.13) determine the location of the stationary point. At this point $\hat{R} = \vec{x}_\sigma$, which, if we introduce the ray concept, tells us that the surface rays leave the surface at the "point of diffraction" tangentially. Equation (C.14) indicates that the stationary point is of second order, so that we need higher-order derivities of the phase. $L^{\sigma\sigma}$, $L^{\beta\sigma}$ and $L^{\beta\beta}$ are coefficients of the second fundamental form of the surface evaluated at the stationary point. They are defined as

$$L^{\sigma\sigma} = \vec{x}_{\sigma\sigma}\cdot\vec{x}_3 \quad , \quad L^{\sigma\beta} = \vec{x}_{\sigma\beta}\cdot\vec{x}_3 \quad , \quad L^{\beta\beta} = \vec{x}_{\beta\beta}\cdot\vec{x}_3$$

Using the relationships given above, one can find the expansion coefficients g_{kl}, a_{mn}, b_{mn} and A_{pq} in (C.3). Zeroth and first-order terms in (C.3) give us formulas (5.6).

A few remarks should be made concerning the expansion presented in (C.3). First of all, (C.3) is a doubly infinite series; therefore, for each fixed power of k^{-1} a finite number of terms should be summed up. The coefficients of various terms in these finite sums, namely A_{pq}'s , become very complicated when p and q are greater than 0 or 1. Another difficulty with this series is that when the stationary point of the phase is of an order higher than 1, the difference between the order of the successive terms (when they are ordered according to the descending power of k) becomes very small, and consequently the infinite series converges very slowly. For instance, in our problem where $\delta = 3$ and $\tau = 2$ (stationary point is of second order), sometimes the difference between the orders of successive terms is $k^{-1/6}$, which indicates the weak convergence (in an asymptotic sense) of the expansion in the cases where the frequency is not very large.

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